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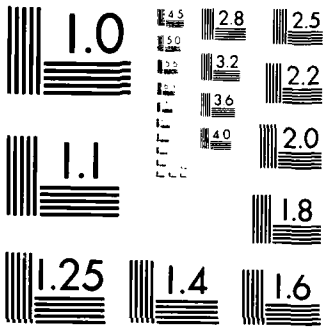

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TECHNICAL REPORT CERC-85-1

METHODS FOR COMPUTING CONFIDENCE INTERVALS FOR SPECTRAL ESTIMATES IN MONTHLY REPORTS OF THE CALIFORNIA COASTAL DATA INFORMATION PROGRAM

by

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Confidence intervals are presented for the spectral estimates from the Coastal Data Information Program. The intervals are based on chi-square values. Tables of the required chi-square values are also presented. Two methods are derived and presented for building confidence intervals for the estimates of the longshore component of radiation stress. The relative merits of both methods are discussed and the more useful of the two is indicated.

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PREFACE

The techniques for computing confidence intervals for spectral estimates from the Coastal Data Information Program were developed at the University of Wyoming during the period from August 1982 to August 1983. This report was done at the request of and was funded by the US Army Engineer Division, South Pacific, as part of the Coastal Field Data Collection Program at the Coastal Engineering Research Center (CERC) of the US Army Engineer Waterways Experiment Station (WES). The work was performed by Drs. Michael E. Andrew, Research Statistician, CERC, and Leon E. Borgman, Professor of Statistics, University of Wyoming.

This report was revised by Dr. Andrew and edited for publication at CERC under the general supervision of Drs. Dennis R. Smith, Chief, Prototype Measurement and Analysis Branch, William L. Wood, Chief, Engineering Development Division, and Dr. Robert W. Whalin, Chief, CERC.

Commanders and Directors of WES during the conduct of this study and preparation and publication of this report were COL Tilford C. Creel, CE, and COL Robert C. Lee, CE. Technical Director was Mr. Frederick R. Brown.

CONTENTS

METHODS FOR COMPUTING CONFIDENCE INTERVALS FOR SPECTRAL ESTIMATES IN MONTHLY REPORTS OF THE CALIFORNIA COASTAL DATA INFORMATION PROGRAM

PART 1: INTRODUCTION

The limits containing a certain quantity with a probability of $1-\alpha$ are called the $(1-\alpha) \cdot 100$ percent confidence limits for that specific quantity. The interval between the confidence limits is called a confidence interval. The purpose of this paper is to present methods for finding confidence intervals for the quantities estimated in the monthly reports of the Coastal Data Information Program sponsored by the US Army Corps of Engineers and the State of California Department of Boating and Waterways. The theory presented here pertaining to confidence limits for estimates of the spectral energy has been in existence for some time and has been verified. A good discussion of its validity appears in a recent paper by Donelan and Pierson (1983). The theory concerning the probability law for the longshore component of radiation stress estimates and associated confidence intervals (as well as the approximate confidence intervals) is relatively new and has not been widely discussed in any applied sense. However, some theoretical results have been presented by N. R. Goodman (1957) on the probability law for estimates of the cospectrum.

PART 2: CONFIDENCE INTERVAL FOR PERCENT ENERGY ESTIMATES

It has been shown that, under the assumption of a stationary Gaussian sea surface, the normalized spectral density estimates obtained by finite Fourier techniques and frequency smoothing have a chi-square divided by degrees of freedom probability distribution (Borgman 1973). Furthermore, this theory has recently been demonstrated to yield accurate estimates of the sampling variability for the spectral estimates (Donelan and Pierson 1983).

If $\hat{S}(f)$ denotes the spectral estimate from an average of k/2 raw spectral lines centered at frequency f, then

$$\frac{\hat{S}(f)}{S(f)} = \frac{\chi_k^2}{k} \tag{2.1}$$

where χ_k^2 represents a chi-square random variable with k degrees of freedom. Thus

$$\Pr[\chi^2_{k,\alpha/2} < \frac{\hat{S}(f)k}{S(f)} < \chi^2_{k,1-\alpha/2}] = 1-\alpha \tag{2.2}$$

where $\chi^2_{k,a}$ is the value of the chi-square variable for which

$$\Pr[\chi_k^2 < \chi^2_{k,a}] = a \tag{2.3}$$

The estimates provided in the reports are in terms of percent of the total energy. Thus it is necessary to find a relation between $\hat{S}(f)$ and $\hat{PE}(f)$, the percent of total energy for the frequency interval centered at f.

$$\hat{PE}(f) = (\Delta f \sum_m \tilde{S}_m) \frac{100}{TE} \tag{2.4}$$

where

Δf = digital increment in the frequency domain

$\tilde{S}_m$ = raw spectral line for frequency $m\Delta f$

TE = total energy in spectrum (2.5)

Note that the summation over m refers only to that frequency interval centered at f. Thus,

$$\hat{PE}(f) = \frac{k}{2} \Delta f \, \hat{S}(f) \frac{100}{TE} \tag{2.6}$$

since

$$\hat{S}(f) = \frac{2}{k} \sum_m \tilde{S}_m \tag{2.7}$$

Then

$$\Pr\left[\frac{\Delta f}{2} \frac{100}{TE} \chi^2_{k,\alpha/2} < \frac{\hat{PE}(f)}{S(f)} < \frac{\Delta f}{2} \frac{100}{TE} \chi^2_{k,1-\alpha/2}\right] = 1-\alpha \tag{2.8}$$

or

$$\Pr\left[\frac{\hat{PE}(f)}{\Delta f \; 100} \frac{2TE}{\chi^2_{k,1-\alpha/2}} < S(f) < \frac{\hat{PE}(f)}{\Delta f \; 100} \frac{2TE}{\chi^2_{k,\alpha/2}}\right] = 1-\alpha \tag{2.9}$$

Thus,

$$\left(\frac{\hat{PE}(f)}{\Delta f \; 100} \frac{2TE}{\chi^2_{k,1-\alpha/2}} , \frac{\hat{PE}(f)}{\Delta f \; 100} \frac{2TE}{\chi^2_{k,\alpha/2}}\right) \tag{2.10}$$

yields a $(1-\alpha) \times 100$ percent confidence interval for S(f). However, since the reports are in terms of percent of total energy, consider the following expression:

$$\Pr\left[\hat{PE}(f) \frac{k}{\chi^2_{k,1-\alpha/2}} < \frac{k\Delta f}{2} \times \frac{S(f)}{TE} \times 100 < \hat{PE}(f) \frac{k}{\chi^2_{k,\alpha/2}}\right] = 1-\alpha \tag{2.11}$$

Then

$$\left[\hat{PE}(f)\ \frac{k}{\chi^2_{k,1-\alpha/2}}\ ,\quad \hat{PE}(f)\ \frac{k}{\chi^2_{k,\alpha/2}}\right] \tag{2.12}$$

is a $(1-\alpha) \cdot 100$ percent confidence interval for the quantity

$$\frac{k\Delta f}{2}\ \frac{S(f)}{TE}\ 100 \tag{2.13}$$

The quanitity

$$PE(f) = \frac{k\Delta f}{2}\ \frac{S(f)}{TE} \cdot 100 \tag{2.14}$$

represents the discrete approximation of the energy in the frequency interval centered at f containing k/2 raw spectral lines.

The monthly reports for the Coastal Data Information Program contain nine spectral estimates for each location and time. The spectral estimates are given by period bands and result from varying values of k, as shown in Table 1. Also listed are the values for $\chi^2_{k,\alpha/2}$ and $\chi^2_{k,1-\alpha/2}$ for the appropriate values of k by period band.

Table 1

Coastal Data Information Program Spectral Estimates

Period	k	$\chi^2_{k,.05}$	$\chi^2_{k,.95}$
22+	92	71.76023	116.50839
22-18	20	10.84948	31.41630
18-16	16	7.96032	26.30112
16-14	18	9.38799	28.87667
14-12	24	13.84490	36.42149
12-10	34	21.66101	48.61088
10- 8	52	36.43630	69.82803
8- 6	84	63.87607	106.39197
6- 4	172	142.6713	203.59982

Table 2 lists the estimates from the Crescent City array for May 1, 1981 at time 2120. Also listed are the lower and upper 90% confidence bounds for the quantity PE (see Appendix B for a copy of the original pages from the report).

Table 2

Crescent City Spectral Estimates

Period	$\hat{PE}$	Lower	Upper
22+	3.0	2.369	3.846
22-18	0.6	.382	1.106
18-16	0.6	.365	1.206
16-14	3.0	1.870	5.752
14-12	43.8	28.862	75.927
12-10	20.7	14.478	32.492
10- 8	12.3	9.160	17.554
8- 6	7.4	5.842	9.731
6- 4	9.1	7.688	10.971

Using the values in Table 1 and Equation (1.2) to obtain confidence intervals on the quantity defined as PE is relatively simple for any set of estimates taken from the monthly reports.

PART 3: PROBABILITY LAW FOR THE LONGSHORE COMPONENT OF RADIATION STRESS

The longshore component of radiation stress denoted by S_{xy} has been shown to be a function of the cospectrum of the offshore and longshore surface slopes (Seymour and Higgins 1978). Assuming linear wave theory, it is possible to determine the probability law for S_{xy} in a closed mathematical form. Denote the two surface slope time series by

$$\begin{aligned} \eta_x(n\Delta t) &= \text{offshore slope} \\ \eta_y(n\Delta t) &= \text{longshore slope} \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} U_x(m\Delta f) - iV_x(m\Delta f) &= \Delta t \sum_{n=0}^{N-1} \eta_x(n\Delta t) \exp(-i2\pi mn/N) \\ U_y(m\Delta f) - iV_y(m\Delta f) &= \Delta t \sum_{n=0}^{N-1} \eta_y(n\Delta t) \exp(-i2\pi mn/N) \end{aligned} \tag{3.2}$$

which are the discrete Fourier transforms of the two slope components. The estimate for the cospectrum of η_x with η_y is

$$\tilde{C}_{xy}(m\Delta f) = \frac{U_x(m\Delta f)\ U_y(m\Delta f) + V_x(m\Delta f)\ V_y(m\Delta f))}{N\Delta t} \tag{3.3}$$

An estimate for S_{xy} as a function of frequency is

$$\tilde{S}_{xy}(m\Delta f) = \frac{pg\ n(k)}{k^2}\ \tilde{C}_{xy}(m\Delta f) \tag{3.4}$$

where

k = wave number

p = water density

g = acceleration due to gravity

$$n(k) = \frac{1}{2} + \frac{k\ h}{\sinh\ (2kh)} \tag{3.5}$$

h = measured average depth

and n(k) and k are functions of frequency according to the following relation:

$$(2\pi m \cdot f)^2 = gk \tanh(kh) \tag{3.6}$$

It can be shown that $U_x(m \cdot f)$, $U_y(m \cdot f)$, $V_x(m \cdot f)$, $V_y(m \cdot f)$ are distributed multivariate gaussian with mean zero and variance covariance matrix:

$$C = \frac{N \cdot t}{2} \begin{bmatrix} S_{xx}(m \cdot f) & C_{xy}(m \cdot f) & 0 & 0 \\ C_{xy}(m \cdot f) & S_{yy}(m \cdot f) & 0 & 0 \\ 0 & 0 & S_{xx}(m \cdot f) & C_{xy}(m \cdot f) \\ 0 & 0 & C_{xy}(m \cdot f) & S_{yy}(m \cdot f) \end{bmatrix} \tag{3.7}$$

where

$S(f)$ = The spectral density of the sea surface

$S_{xx}(f)$ = The spectral density of the offshore slope

$S_{yy}(f)$ = The spectral density of the longshore slope

Also the four random variables are independent for different values of m in the interval $0 < m < N/2$.

In order to obtain the probability law for $C_{xy}(m \cdot f)$ it is necessary to form the characteristic function. Define the characteristic function for a random variable zl to be

$$\Phi_{zl}(t) = E[e^{itzl}] \tag{3.8}$$

where E stands for statistical expectation and $i = \sqrt{-1.0}$. Thus if we let $zl = C_{xy}(m \cdot f)$ then

$$\begin{aligned} \Phi_{zl}(t) &= E[e^{itzl}] \\ &= E\left(\exp\left\{it\left[U_x(m \cdot f)\, U_y(m \cdot f) + V_x(m \cdot f)\, V_y(m \cdot f)\right]\right\}\right) \\ &= E(\exp it\, U^{(m)}) , \end{aligned} \tag{3.9}$$

Then let

$$= \frac{\cos \quad \sin}{\cos \quad \sin} , \tag{4.22}$$

and then

$$\frac{S_{xy}(f)}{S_{xy}(f)} \quad \frac{E(f)}{E(f)} = \frac{S(f)}{S(f)} \tag{4.23}$$

It has already been stated that S(f)/S(f) has a chi-square divided by degrees of freedom probability distribution. Then for $\hat{S}_{xy}(f) > 0$

$$\Pr\left[\frac{\chi^2_{k,\alpha/2}}{k} \quad \frac{\hat{S}_{xy}(f)}{S_{xy}(f)} \quad \frac{\chi^2_{k,1-\alpha/2}}{k} \right] = 1-\alpha \tag{4.24}$$

or

$$\Pr\left[\frac{k\ \hat{S}_{xy}(f)}{\chi^2_{k,1-\alpha/2}} \quad S_{xy}(f) \quad \frac{k\ \hat{S}_{xy}(f)}{\chi^2_{k,\alpha/2}} \right] = 1-\alpha \tag{4.25}$$

Note that if $S_{xy}(f) = 0.0$ then the inequalities are reversed. Thus the resulting confidence bounds are reversed.

The expression for can be rewritten by letting $\Delta = \theta - \hat{\theta}$ and

$$\begin{aligned} &= \frac{\sin 2}{\sin 2} \\ &= \frac{\sin 2}{\sin 2(\ +\ )} \\ &= \frac{\sin 2}{\sin 2 \quad \cos 2 + \cos 2 \quad \sin 2} \\ &= 1/\ * \end{aligned} \tag{4.26}$$

where $* = \cos 2 + \sin 2 \cot(2\ )$. Then the confidence interval can be written as follows:

several different values of the frequency averaging bandwidth then plotting the coherence estimate vs. bandwidth. The coherence estimate will be equal to one for one spectral line per averaging band. It will then drop off rapidly toward zero for increasing bandwidth until it reaches a plateau. Beyond this plateau it will again drop off to zero with increasing number of lines per average. This is due to the smoothing across features in the true spectral densities. The largest bandwidth that will keep the coherence on the plateau is a good choice since $\hat{coh}(f)$ is stabilizing around its theoretical value. Also, the largest bandwidth on the plateau provides the highest degrees of freedom for the estimates (Borgman 1973). Estimates for coh(f) are not available in the monthly reports. Due to this difficulty it is not possible to obtain approximate values for t_1 and t_2. However it is possible, under certain assumptions, to obtain rough confidence intervals for $S_{xy}(f)$. The monthly reports contain values for the apparent angle θ which is related to $S_{xy}(f)$ by the following equation:

$$S_{xy}(f) = E(f)\ n(f)\ \cos\theta\ \sin\theta \tag{4.19}$$

where E(f) is the energy contained in the frequency interval centered at f (Seymour and Higgins 1977).

Let E[f] be approximated by

$$E(f) = S(f)\ \delta(f) \tag{4.20}$$

where $\delta(f)$ is the width of the frequency interval centered at f , in this case $\delta(f) = \Delta f\ k/2$. Also let E[f] be estimated by

$$\hat{E}(f) = \hat{S}(f)\ \delta(f) \tag{4.21}$$

However, the function $f^*(z)$ contains an unknown parameter D^2 where

$$D^2 = \frac{S_{xx}(f)\ S_{yy}(f) - C^2_{xy}(f)}{C^2_{xy}(f)}$$

$$= \frac{S_{xx}(f)\ S_{yy}(f)}{C^2_{xy}(f)} - 1.0$$

$$= \mathrm{coh}(f)^{-1} - 1.0 \tag{4.15}$$

The expression coh(f) stands for the coherence of the x slope with the y slope. Coherence is defined to be

$$\mathrm{coh}(f) = \frac{C^2_{xy}(f) + q^2_{xy}(f)}{S_{xx}(f)\ S_{yy}(f)} \tag{4.16}$$

or the modulus squared of the cross spectrum divided by the two spectral densities. In the case of x slope and y slope the quad spectrum $q_{xy}(f)$ is theoretically zero, so

$$\mathrm{coh}(f) = \frac{C^2_{xy}(f)}{S_{xx}(f)\ S_{yy}(f)} \tag{4.17}$$

Since the coherence is not known, it is not possible to determine D^2 and thus t_1 and t_2 exactly. If t_1 and t_2 are found to satisfy the two conditions using an estimated value of coh(f), it is not certain that the confidence interval will then have the specified confidence coefficient $(1-\alpha) \cdot 100$ percent. The estimated coherence has the following form:

$$\widehat{\mathrm{coh}}(f) = \frac{\hat{C}^2_{xy}(f)}{S_{xx}(f)\ S_{yy}(f)} \tag{4.18}$$

To minimize any sources of error due to the use of an estimated coherence in obtaining values for t_1 and t_2, it is desirable to have the best possible estimate for coh(f). This is done by computing the coherence estimate for

Case 2: Assume $t_2 \leq 0$, then the result is the same as in Case 1.

Case 3: Assume $t_1 \leq 0$, $t_2 \geq 0$, and $\hat{S}_{xy}(f) \geq 0$, then

$$\Pr[t_1 \leq \frac{\hat{S}_{xy}(f)}{S_{xy}(f)} < t_2] =$$

$$= \Pr[t_1 \leq \frac{\hat{S}_{xy}(f)}{S_{xy}(f)} < 0] + \Pr[0 \leq \frac{\hat{S}_{xy}(f)}{S_{xy}(f)} \leq t_2]$$

$$= \Pr[-\infty < S_{xy}(f) \leq \frac{\hat{S}_{xy}(f)}{t_1}] + \Pr[\frac{\hat{S}_{xy}(f)}{t_2} \leq S_{xy}(f) < \infty] \tag{4.10}$$

$$= 1-\alpha$$

Then for $t_1 \leq 0$, $t_2 \geq 0$ and $\hat{S}_{xy}(f) \geq 0$, $(1-\alpha) \cdot 100$ percent confidence interval is

$$(-\infty, \frac{\hat{S}_{xy}(f)}{t_1}) \text{ and } (\frac{\hat{S}_{xy}(f)}{t_2}, \infty) \tag{4.11}$$

Case 4: If $t_1 \leq 0$, $t_2 \leq 0$ and $\hat{S}_{xy}(f) < 0$, then the intervals become

$$(-\infty, \frac{\hat{S}_{xy}(f)}{t_2}) \text{ and } (\frac{\hat{S}_{xy}(f)}{t_1}, \infty) \tag{4.12}$$

The major difficulty in using the above intervals is in the determination of the values for t_1 and t_2. The conditions for this are

$$\int_{t_2}^{\infty} f^*(z)dz = \alpha/2 \tag{4.13}$$

$$\int_{-\infty}^{t_1} f^*(z)dz = \alpha/2 \tag{4.14}$$

$$f^*(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q_z(t)\, e^{-itz} dz$$

$$= \frac{1}{2\pi} \left(\frac{\nu}{D}\right)^{2\nu} \int_{-\infty}^{\infty} \frac{e^{-itz}}{\{t-ia\}^{\nu}(t-ib)^{\nu}}\, dt$$

$$= \left(\frac{\nu}{D}\right)^{2\nu} f(z)$$

$$= \left(\frac{\nu}{D}\right)^{2\nu} \frac{e^{\alpha z}}{(\nu-1)!} \sum_{m=0}^{\nu-1} \frac{(\nu+m-1)!}{(\nu-m-1)!} \frac{z^{\nu-m-1}}{m!} \frac{1}{(a+b)^{\nu+m}} \tag{4.6}$$

and

$$\alpha = a \ , \ z < 0$$

$$\alpha = b \ , \ z \geq 0$$

Now having obtained $f^*(z)$ it is possible to find t_1 and t_2 such that

$$\Pr\left[t_1 \leq \frac{\hat{S}_{xy}(f)}{S_{xy}(f)} \leq t_2\right] = 1-\alpha \tag{4.7}$$

In order to invert inequality, it is necessary to break the values of t_1 and t_2 into cases.

Case 1: Assume $t_1 \geq 0$, then

$$\Pr\left[\frac{\hat{S}_{xy}(f)}{t_2} \leq S_{xy}(f) \leq \frac{\hat{S}_{xy}(f)}{t_1}\right] = 1-\alpha \tag{4.8}$$

Then the $(1-\alpha) \cdot 100$ percent confidence interval for $S_{xy}(f)$ is

$$\left(\frac{\hat{S}_{xy}(f)}{t_2}, \frac{\hat{S}_{xy}(f)}{t_1}\right) \tag{4.9}$$

PART 4: A CONFIDENCE INTERVAL FOR THE LONGSHORE COMPONENT OF RADIATION STRESS

In the section of this report where the probability law for the longshore component of radiation stress was developed, the random variable z, where $z = \nu \hat{S}_{xy}(f)/\beta(f)$, was obtained. Here consider the variable

$$z = \hat{S}_{xy}(f)/S_{xy}(f) \tag{4.1}$$

where $\hat{S}_{xy}(f)$ is the estimate defined in the previous section, and $S_{xy}(f) = \alpha(f)$ is the theoretical value. The characteristic function for the new variable z is from equation 2.28.

$$Q_z(t) = \{1 - \frac{it}{\nu} + \frac{t^2 D^2}{\nu^2}\}^{-\nu} \tag{4.2}$$

where

$$D^2 = \frac{S_{xx}(f)\ S_{yy}(f) - C_{xy}^2(f)}{C_{xy}^2(f)} \tag{4.3}$$

The function $Q_z(t)$ can be rewritten

$$Q_z(t) = (\frac{\nu}{D})^{2\nu}\ \{t-ia\}^{-\nu}\{t-ib\}^{-\nu} \tag{4.4}$$

where

$$\begin{aligned} a &= \frac{\nu}{2D}\ (\frac{1}{D} + \sqrt{1/D^2 + 4}) \\ b &= \frac{\nu}{2D}\ (\frac{1}{D} - \sqrt{1/D^2 + 4}) \end{aligned} \tag{4.5}$$

Since $(\frac{\nu}{D})^{2\nu}$ is constant, it is convenient to obtain the probability density of z from the results in the previous section. Thus

Case 2: $z < 0$

If Γ is chosen to be $t = Re^{i\theta}$ for $0 \le \theta \le \pi$, then the conditions are met (see Appendix A for proof) and

$$\int_{-\infty}^{\infty} e^{-itz}\, Q_z(t)dt = 2\pi i\; a_{-1} \tag{3.51}$$

where

$$a_{-1} = \lim_{t \to ia} \frac{1}{(\nu-1)!} \frac{\partial^{\nu-1}}{\partial t^{\nu-1}} \{(t-ia)^{\nu} e^{-itz}\, Q_z(t)\} \tag{3.52}$$

As in case 1, a_{-1} can be found in closed form and is

$$a_{-1} = \frac{e^{az}}{(\nu-1)!} \sum_{m=0}^{\nu-1} \frac{(\nu+m-1)!}{(\nu-m-1)!} \frac{(-1)^{\nu+m} i}{m!} \frac{z^{\nu-m-1}}{(a-b)^{\nu+m}} \tag{3.53}$$

and

$$\begin{aligned} f(z) &= \frac{1}{2\pi} 2\pi i\; a_{-1} \\ &= \frac{e^{az}}{(\nu-1)!} \sum_{m=0}^{\nu-1} \frac{(\nu+m-1)!}{(\nu-m-1)!} \frac{|z|^{\nu-m-1}}{m!} \frac{1}{(a-b)^{\nu+m}} \end{aligned} \tag{3.54}$$

In general,

$$f(z) = \frac{e^{\alpha z}}{(\nu-1)!} \sum_{m=0}^{\nu-1} \frac{(\nu+m-1)!}{(\nu-m-1)!} \frac{|z|^{\nu-m-1}}{m!} \frac{1}{(a-b)^{\nu+m}} \tag{3.55}$$

where

$\alpha = a \qquad z > 0$

$\alpha = b \qquad z < 0$

Case 1: $z \geq 0$.

If for the curve Γ we choose $t = Re^{-i\theta}$ $0 \leq \theta \leq \pi$, then the conditions for using the residue theorem are fulfilled (see Appendix A for proof), and

$$\int_{-\infty}^{\infty} e^{-itz}\, Q_z(t)dt = -2\pi i\, a_{-1} \tag{3.44}$$

where

$$a_{-1} = \lim_{t\to ib} \frac{1}{(\nu-1)!} \frac{\partial^{\nu-1}}{\partial t^{\nu-1}} \{(t-ib)^{\nu} e^{-itz}\, Q_z(t)\} \tag{3.45}$$

Since $Q_z(t) = (t-ia)^{-\nu}(t-ib)^{-\nu}$

then

$$a_{-1} = \lim_{t\to ib} \frac{1}{(\nu-1)!} \frac{\partial^{\nu-1}}{\partial t^{\nu-1}} \{e^{-itz}\, (t-ia)^{-\nu}\} \tag{3.46}$$

The derivative can be evaluated using the formula in equation (3.47).

For any two functions $f(x)$ and $g(x)$ such that the required higher order derivatives exist, it is possible to show that (Abramowitz and Stegun 1970)

$$\frac{\partial^n\, f\cdot g}{\partial x^n} = \sum_{k=0}^{n} \binom{n}{k} \frac{\partial^{n-k} f}{\partial x^{n-k}} \frac{\partial g^k}{\partial x^k} \tag{3.47}$$

$$\frac{\partial^{\nu-1}\, e^{-itz}\, (t-ib)^{-\nu}}{\partial t^{\nu-1}} = \sum_{m=0}^{\nu-1} \binom{\nu-1}{m} (-iz)^{\nu-m-1}\, e^{-itz} (-1)^m \frac{(\nu+m-1)!}{(\nu-1)!} (t-ib)^{-(\nu+m)} \tag{3.48}$$

Thus

$$a_{-1} = \frac{e^{bz}}{(\nu-1)!} \sum_{m=0}^{\nu-1} \frac{(\nu+m-1)!}{(\nu-m-1)!} \frac{(-1)^{\nu+m}}{m!}\, i\, \frac{z^{\nu-m-1}}{(b-a)^{+(\nu+m)}} \tag{3.49}$$

Then for $z \cdot 0$

$$f(t) = -\frac{1}{2\pi}\, 2\pi i\, a_{-1}$$

$$= \frac{e^{bz}}{(\nu-1)!} \sum_{m=0}^{\nu-1} \frac{(\nu+m-1)!}{(\nu-m-1)!} \frac{z^{\nu-m-1}}{m!} \frac{1}{(a-b)^{\nu+m}} \tag{3.50}$$

$$\int_c e^{-itz}\, Q_z(t)dt = \int_\Gamma e^{-itz}\, Q_z(t)dt + \int_L e^{-itz}\, Q_z(t)dt \tag{3.38}$$

where Γ is the semicircle part of c and L is the line segment. Now if it can be shown that

$$\lim_{R\to\infty} \int_\Gamma e^{-itz}\, Q_z(t)dt = 0 \tag{3.39}$$

then

$$\lim_{R\to\infty} \int_c e^{-itz} Q_z(t)dt = \int_{-\infty}^{\infty} e^{-itz}\, Q_z(t)dt \tag{3.40}$$

However, by the Residue Theorem of complex analysis (Mardsen 1973), the integral

$$\lim_{R\to\infty} \int_c e^{-itz}\, Q_z(t)dt = \pm\, 2\pi i\, a_{-1} \tag{3.41}$$

where a_{-1} is the residue of the function $e^{-itz}\, Q_z(t)$ at the singularity contained in the curve c. The function $e^{-itz}\, Q_z(t)$ has a singularity at ia and one at ib. Both of these points are said to be poles of order ν. Thus the function $e^{-itz}\, Q_z(t)$ has residues equal to (Spiegel 1964)

$$\lim_{t\to t_0} \frac{1}{(\nu-1)!} \frac{\partial^{\nu-1}}{\partial t^{\nu-1}} \{(t-t_0)^\nu e^{-itz}\, Q_z(t)\} \tag{3.42}$$

where t_0 = ia and ib. If the residue is in the upper half of the plane, then the integral has value $+2\pi i\, a_{-1}$. If it is in the lower half, then the integral is $-2\pi i\, a_{-1}$

It is necessary to consider two cases in order to compute the integral

$$\int_{-\infty}^{\infty} e^{-itz}\, Q_z(t)dt \tag{3.43}$$

If the formula is factored inside of the brackets, the function becomes

$$Q_z(t) = (t-ia)^{-\nu}(t-ib)^{-\nu} \tag{3.32}$$

where

$$a = \frac{\gamma(f) + \sqrt{\gamma(f)^2 + 4}}{2} \tag{3.33}$$

$$b = \frac{\gamma(f) - \sqrt{\gamma(f)^2 + 4}}{2} \tag{3.34}$$

According to the inversion theorem for characteristic function (Rao, 1973), the probability density function of the variable $z = \nu\hat{S}_{xy}(f)/\beta(f)$ is

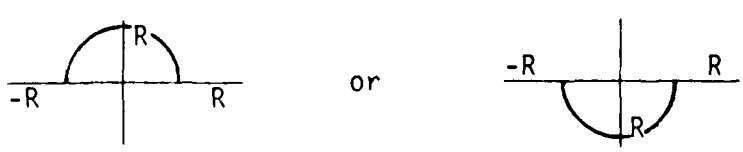
$$f(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itz}\, Q_z(t)dt \tag{3.35}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itz}\, (t-ia)^{-\nu}(t-ib)^{-\nu}\, dt \tag{3.36}$$

It is necessary to make use of the theory of contour integrals in order to solve for f(z). Consider the integral on the curve c

$$\int_c e^{-itz}\, Q_z(t)dt \tag{3.37}$$

where the curve c is either of the semicircles of radius R centered at the origin and the line segment from -R to R, as indicated below.



Since the curve c is closed and since $Q_z(t)$ is analytic everywhere but at the points ia and ib, then

$$\beta(m\Delta f) = pg \frac{n(k)}{k^2} (S_{xx}(m\Delta f)\ S_{yy}(m\Delta f) - C_{xy}{}^2(m\Delta f))^{\frac{1}{2}} \tag{3.24}$$

Since $\frac{n(k)}{k^2} \hat{C}_{xy}(m\Delta f)$ is independent for different values of m it is possible to use another property of characteristic function. That is, if

$$z = \sum_m z_m \tag{3.25}$$

where z_m is independent for different m, then

$$Q_z(t) = \{Q_z(t)\}^m \tag{3.26}$$

Under the assumption that the parameters $\alpha(m\Delta f)$ and $\beta(m\Delta f)$ can be taken to be constant over the interval of frequency smoothing if

$$z3 = \frac{pg}{\nu} \sum_m \frac{n(k)}{k^2} \tilde{C}_{xy}(m\Delta f) \ , \tag{3.27}$$

then

$$Q_{z3}(t) = \{1 - it \frac{\alpha(f)}{\nu} + t^2 \frac{\beta^2(f)}{\nu^2}\}^{-\nu} \tag{3.28}$$

In the last equation $\alpha(f)$ and $\beta(f)$ denote the values of the two parameters at the midpoint of the frequency interval of smoothing.

In order to simplify the function ϕ , if the new random variable z is to be defined as

$$z = \hat{S}_{xy}(f)/\beta(f) \tag{3.29}$$

then

$$Q_z(t) = \{1 - it \frac{\alpha(f)}{\beta(f)} + t^2\}^{-\nu} \tag{3.30}$$

Let $\gamma(f) = \alpha(f)/\beta(f)$ and $\beta(f) \neq 0$. Then Q_z is a function of only two parameters γ and $\beta(f)$ that is

$$Q_z(t) = \{1 - i\gamma(f)t + t^2\}^{-\nu} \tag{3.31}$$

Thus

$$
\begin{aligned}
Q_{z1}(t) &= |I - 2it\ CM|^{-\frac{1}{2}} \\
&= \{|C^*|^2\}^{-\frac{1}{2}} \\
&= |C^*|^{-1} \\
&= \{1 - it\ C_{xy}(m\Delta f) + \frac{t^2}{4}\,(S_{xx}(m\Delta f)\ S_{yy}(m\Delta f) - C_{xy}{}^2(m\Delta f))\}^{-1}
\end{aligned}
\tag{3.18}
$$

If $S_{xy}(m\Delta f)$ is smoothed, then the smoothed estimate will have the following form:

$$\hat{S}_{xy}(f) = \frac{1}{\nu} \sum_m \tilde{S}_{xy}(m\Delta f) \tag{3.19}$$

where the sum is over a specified frequency band with ν spectral lines. Since $\tilde{S}_{xy}(m\Delta f)$ is a function of $\tilde{C}_{xy}(m\Delta f)$, then

$$\hat{S}_{xy}(f) = \frac{pg}{\nu} \sum_m \frac{n(k)}{k^2}\, \tilde{C}_{xy}(m\Delta f) \tag{3.20}$$

One of the properties of characteristic functions is that

$$Q_{\frac{az}{b}}(t) = Q_z(\frac{at}{b}) \tag{3.21}$$

Thus if $z2 = pg\ \frac{n(k)}{k^2}\ \tilde{C}_{xy}(m\Delta f)$, then

$$
\begin{aligned}
Q_{z2}(t) &= \{1 - it\ pg\ \frac{n(k)}{k^2}\ C_{xy}(m\Delta f) \\
&\quad + \frac{t^2}{4}\ pg\ \frac{n(k)^2}{k^4}\ (S_{xx}(m\Delta f)\ S_{yy}(m\Delta f) - C_{xy}{}^2(m\Delta f))\}^{-1} \\
&= \{1 - it\ \alpha(m\Delta f) + t^2\beta^2(m\Delta f)\}^{-1}
\end{aligned}
\tag{3.22}
$$

and

$$\alpha(m\Delta f) = pg\ \frac{n(k)}{k^2}\ C_{xy}(m\Delta f) \tag{3.23}$$

Now

$$2it\ CM = \frac{it}{2}\begin{pmatrix} C_{xy}(m\Delta f) & S_{xx}(m\Delta f) & 0 & 0 \\ S_{yy}(m\Delta f) & C_{xy}(m\Delta f) & 0 & 0 \\ 0 & 0 & C_{xy}(m\Delta f) & S_{xx}(m\Delta f) \\ 0 & 0 & S_{yy}(m\Delta f) & C_{xy}(m\Delta f) \end{pmatrix} \tag{3.13}$$

$$I - 2it\ CM = \begin{pmatrix} C^* & 0 \\ 0 & C^* \end{pmatrix} \tag{3.14}$$

where

$$C^* = \begin{pmatrix} 1 - \frac{it}{2} C_{xy}(m\Delta f) & -\frac{it}{2} S_{xx}(m\Delta f) \\ -\frac{it}{2} S_{yy}(m\Delta f) & I - \frac{it}{2} C_{xy}(m\Delta f) \end{pmatrix} \tag{3.15}$$

Then

$$\begin{aligned} |I - 2it\ CM| &= \left|\begin{pmatrix} C^* & 0 \\ 0 & C^* \end{pmatrix}\right| \\ &= |C^*|\ |C^*| \\ &= |C^*|^2 \end{aligned} \tag{3.16}$$

$$\begin{aligned} |C^*| &= (1 - \frac{it}{2} C_{xy}(m\Delta f))^2 + \frac{i^2t^2}{4} S_{xx}(m\Delta f)\ S_{yy}(m\Delta f) \\ &= 1 - it\ C_{xy}(m\Delta f) - \frac{i^2t^2}{4} C_{xy}^2(m\Delta f) + \frac{i^2t^2}{4} S_{xx}(m\Delta f)\ S_{yy}(m\Delta f) \\ &= 1 - it\ C_{xy}(m\Delta f) + \frac{t^2}{2} (S_{xx}(m\Delta f)\ S_{yy}(m\Delta f) - C_{xy}^2(m\Delta f)) \end{aligned} \tag{3.17}$$

where

$$\underline{U}' = [U_x(m\Delta f),\ U_y(m\Delta f),\ V_x(m\Delta f),\ V_y(m\Delta f)]$$

and

$$m = \frac{1}{N\Delta t}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \frac{1}{2}$$

Since $f_{\underline{U}}(\underline{U}) = \frac{1}{(2\pi)^2|C|^{\frac{1}{2}}}\, e^{-\frac{1}{2}\underline{U}'C^{-1}\underline{U}}$ (3.10)

$$Q_{z1}(t) = \int_{-\infty}^{\infty} e^{it\ \underline{U}'M\underline{U}} \cdot \frac{1}{(2\pi)^2|C|^{\frac{1}{2}}}\, e^{-\frac{1}{2}\underline{U}'\underline{C}^{-1}\underline{U}}\, d\underline{U}$$

$$= \int_{-\infty}^{\infty} \frac{1}{(2\pi)^2|C|^{\frac{1}{2}}}\, e^{-\frac{1}{2}(\underline{U}'\underline{C}^{-1}\underline{U}\ -2it\underline{V}'M\underline{U})}\, d\underline{U}$$

$$= \int_{-\infty}^{\infty} \frac{1}{(2\pi)^2|C|^{\frac{1}{2}}}\, e^{-\frac{1}{2}\ \underline{U}'[\underline{C}^{-1}-2it\ M]\underline{U}}\, d\underline{U} \qquad (3.11)$$

$$Q_{z1}(t) = \frac{|C^{-1}+2it\ M|^{-\frac{1}{2}}}{|C|^{\frac{1}{2}}} \int_{-\infty}^{\infty} \frac{1}{(2\pi)^2|C^{-1}+2it\ M|^{-\frac{1}{2}}}\, e^{-\frac{1}{2}\underline{U}'[C^{-1}-2it\ M]\underline{U}}\, d\underline{U}$$

$$= \frac{|C^{-1}+2it\ M|^{-\frac{1}{2}}}{|C|^{\frac{1}{2}}}$$

$$= |C|^{-\frac{1}{2}}\ |C^{-1}-2it\ M|^{-\frac{1}{2}}$$

$$= |I\ -\ 2it\ CM|^{-\frac{1}{2}} \qquad (3.12)$$

$$\left[\frac{\epsilon^* k\, \hat{S}_{xy}(f)}{\chi^2_{k,1-\alpha/2}}\,,\ \frac{\epsilon^*\, k\, \hat{S}_{xy}(f)}{\chi^2_{k,\alpha/2}}\right] \tag{4.27}$$

It is possible now to compute confidence intervals for $S_{xy}(f)$ given arbitrary choice of Δ. If Δ is assumed to be small in absolute value compared to $\cot(2\hat{\theta})$, then ϵ^* will be close to 1.0. However, for $\hat{\theta}$ close to zero $\cot(2\hat{\theta})$ becomes very large and can dominate the value of ϵ^*. In this case it may be desirable to choose a value for ϵ^* that is different from 1.0, according to an arbitrary choice for Δ. Table 3 lists some values of ϵ^* for typical $\hat{\theta}$ values. For negative $\hat{\theta}$ values simply look under $-\Delta$.

Table 3

Values of ϵ^* for Typical $\hat{\theta}$ Values

Δ	3	7	9	11	15	47
-2.0	.33	.72	.78	.82	.88	1.00
-1.0	.67	.86	.89	.91	.94	1.00
- .5	.83	.93	.95	.96	.97	.99
0	1.0	1.0	1.0	1.0	1.0	1.0
.5	1.17	1.07	1.05	1.04	1.03	1.01
1.0	1.33	1.14	1.11	1.09	1.06	1.00
2.0	1.66	1.28	1.21	1.17	1.12	.99

The following estimates were taken from the Crescent City array on May 1, 1981 (time 2120) (Appendix B). Table 4 lists the period bands, associated midpoint frequencies, wave numbers and value for the function n(f). Table 5 lists the reported angles measured clockwise from true north, the associated $\hat{\theta}$ values that are counter-clockwise from beach normal (positive x axis), and the S_{xy} values computed with the following formula:

$$S_{xy}(f) = E(f)\, n(f) \cos\hat{\theta} \sin\hat{\theta} \tag{4.28}$$

Table 4

Midpoint Frequencies and Wave Number Values

Period	Midpoint Frequency	Wave Number	$n(f)$
22+	.00273	.00208	.51580
22-18	.05050	.01027	.99797
18-16	.05903	.01204	.99623
16-14	.06696	.01306	.99377
14-12	.07738	.02412	.98897
12-10	.09167	.03385	.97860
10- 8	.11250	.05098	.95325
8- 6	.14580	.08567	.88238
6- 4	.20833	.17484	.68726

Table 5

Reported and Adjusted Directional Estimates

Period	Reported Angle*	$\hat{\theta}$	$\hat{S}_{xy}(f)$
22+			
22-18	50.9	-9.1	-1.048
18-16	15.0	-45.0	-3.350
16-14	44.4	-15.6	-8.655
14-12	66.9	6.9	57.903
12-10	72.0	12.0	46.173
10- 8	65.9	5.9	13.437
8- 6	64.5	4.5	5.724
6- 4	57.4	-2.6	-3.176

* Angles in degrees.

Table 6 lists the $\hat{S}_{xy}(f)$ values from Table 5 together with the lower and upper confidence bounds for varying guesses of θ. The chi-square values and degrees of freedom are the same as for the spectral estimates (Table 1).

Table 6

Confidence Intervals for S_{xy}

	Period	S_{xy}	*	Lower	Upper
Δ = -2.0	22-18	- 1.048	1.22	- 2.36	- .82
	18-16	- 3.350	1.03	- 6.93	- 2.10
	16-14	- 8.655	1.12	-18.58	- 6.04
	14-12	57.903	.71	27.09	71.24
	12-10	46.173	.83	26.80	60.16
	10- 8	13.437	.66	6.60	12.66
	8- 6	5.724	.56	2.53	4.21
	6- 4	- 3.176	1.77	- 6.78	- 4.75
Δ = -1.0	22-18	- 1.048	1.11	- 2.14	- .74
	18-16	- 3.350	1.02	- 6.87	- 2.08
	16-14	- 8.655	1.06	-17.59	- 5.72
	14-12	57.903	.86	32.15	86.29
	12-10	46.173	.92	29.71	66.68
	10- 8	13.437	.83	8.31	15.91
	8- 6	5.724	.77	3.48	5.80
	6- 4	- 3.176	1.38	- 5.28	- 3.70
Δ = 0.0	22-18	- 1.048	1.0	- 1.93	- .67
	18-16	- 3.350	1.0	- 6.73	- 2.04
	16-14	- 8.655	1.0	-16.59	- 5.39
	14-12	57.903	1.0	38.16	100.34
	12-10	46.173	1.0	32.29	72.48
	10- 8	13.437	1.0	10.01	19.17
	8- 6	5.724	1.0	4.52	7.53
	6- 4	- 3.176	1.0	- 3.83	- 2.68
Δ = 1.0	22-18	- 1.048	.89	- 1.72	- .60
	18-16	- 3.350	.98	- 6.59	- 2.00
	16-14	- 8.655	.94	-15.59	- 5.07
	14-12	57.903	1.14	43.50	114.39
	12-10	46.173	1.08	34.87	78.28
	10- 8	13.437	1.17	11.71	22.43
	8- 6	5.724	1.22	5.51	9.19
	6- 4	- 3.176	.62	- 2.37	- 1.66
Δ = 2.0	22-18	- 1.048	.78	- 1.50	- .52
	18-16	- 3.350	.96	- 6.46	- 1.96
	16-14	- 8.655	.87	-14.43	- 4.69
	14-12	57.903	1.29	49.22	129.44
	12-10	46.173	1.16	37.46	84.08
	10- 8	13.437	1.34	13.41	25.69
	8- 6	5.724	1.44	6.51	10.84
	6- 4	- 3.176	.23	- .88	- .62

As apparent, the intervals can be very unstable for changing values of [illegible]. Some of the intervals do not even contain $\hat{S}_{xy}(f)$. This demonstrates that great care must be taken in the interpretation of such intervals.

However, if one is willing to assume a value for the apparent angle [illegible] and thus a value for [illegible], then this method is straightforward and relies only on the well-known chi-square random variable.

If one is willing to assume a value for the coherence that is consistent with a specific model for the directional spectrum, then it is possible to compute the constants t_1 and t_2 and obtain the confidence interval. The next section of this report will present such an approach.

PART 5: CONFIDENCE INTERVALS FOR THE LONGSHORE COMPONENT OF RADIATION STRESS BY THE WRAPPED NORMAL SPREADING FUNCTION

Assume that the directional spectrum can be represented by the form

$$S(f, \theta) = S(f)\, D(f,\theta) \tag{5.1}$$

where $S(f)$ is the frequency spectrum of the sea surface and $D(f,\theta)$ has the form

$$D(f,\theta) = \frac{\exp[-\frac{1}{2}(\theta-\theta_0)^2/\sigma^2]}{\sqrt{2\pi}\,\sigma} \quad \theta \tag{5.2}$$

where θ_0 and σ are functions of frequency. This functional form is known as the wrapped normal. The function $D(f,\theta)$ has the following Fourier coefficients:

$$a_n = \frac{\exp(-n^2\sigma^2/2)}{\pi} \cos(n\theta_0) \tag{5.3}$$

$$b_n = \frac{\exp(-n^2\sigma^2/2)}{\pi} \sin(n\theta_0) \tag{5.4}$$

It can be shown that in the case of water elevation x slope and y slope that

$$C_{xy}(f) = S(f)k^2\pi\, b_2/2 \tag{5.5}$$

$$S_{xx}(f) = S(f)k^2(1+\pi a_2)/2 \tag{5.6}$$

$$S_{yy}(f) = S(f)k^2(1-\pi a_2)/2 \tag{5.7}$$

(Borgman,Hagan, and Kuik 1982)

The coherence from Equation (4.17) is

$$\mathrm{Coh}(f) = \frac{C^2_{xy}(f)}{S_{xx}(f)\, S_{yy}(f)} \tag{5.8}$$

Substituting the value in (5.5) through (5.7), it is rewritten to be

$$\mathrm{Coh}(f) = \frac{\sin^2(2\theta_o)}{\exp(4\sigma^2)-\cos(2\theta_o)} \qquad (5.9)$$

Thus the coherence can be tabulated for varying values of the mean direction θ_o and the spread parameter σ. As apparent from the values in Table 7 for typical values of θ_o and σ, the coherence varies from 0.0 to around 0.8.

Having this range of values for Coh(f), it is possible to develop tables for the confidence multipliers t_1 and t_2 from Equations (4.6) through (4.15). These tables appear in Appendix C. Appendix D contains the corresponding values of the parameter D from Equation (4.3).

The confidence interval that results from Equation (4.7) was broken into four cases depending upon the values of t_1, t_2, and $\hat{S}_{xy}(f)$. This makes the interpretation of these intervals difficult since cases 3 and 4 result in intervals of infinite length. However, another look at Equation (4.7) yields the confidence interval on the reciprocal of $S_{xy}(f)$, that is,

$$\left(\frac{t_1}{S_{xy}(f)}, \frac{t_2}{S_{xy}(f)} \right) \qquad (5.10)$$

for $S_{xy}(f) > 0$ and

$$\left(\frac{t_2}{S_{xy}(f)}, \frac{t_1}{S_{xy}(f)} \right) \qquad (5.11)$$

for $S_{xy}(f) < 0$

The confidence interval on $1/S_{xy}(f)$ is more easily computed and exists in a meaningful form for all t_1, t_2, and $\hat{S}_{xy}(f)$ except $\hat{S}_{xy}(f) = 0.0$. Another good property of this interval is the fact that it has a finite

width that is

$$w = \left| \frac{t_2}{\hat{S}_{xy}(f)} - \frac{t_1}{\hat{S}_{xy}(f)} \right|$$

$$= \left| \frac{1}{\hat{S}_{xy}(f)} \right| (t_2 - t_1) \tag{5.12}$$

since $t_2 > t_1$ for all t_1 and t_2 satisfying Equation (4.7).

The relative width for the intervals (5.10) and (5.11) is defined to be

$$RW = (t_2 - t_1) \tag{5.13}$$

The tables in Appendix C also contain the corresponding values of RW. In Appendix D is a listing of the computer programs that compute t_1 and t_2 along with a description of how they can be made operational on any computer equipped with any of the standard FORTRAN languages.

The confidence intervals on $1.0/S_{xy}(f)$ for the values in Table 5 appear in Table 8 along with W , the interval width. A spread parameter of σ = 20 degrees was used with $\theta_o = 0$ in order to compute coh(f) and thus t_1 and t_2 . Equations (5.10), (5.11), and (5.12) were used to obtain the values in Table 8. The values used for t_1 and t_2 were obtained from Appendix C.

Table 7

Coherence as a Function of the Wrapped Normal Parameters

	SIGMA					
THETA	15.0	16.0	17.0	18.0	19.0	20.0
0.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.	.0152	.0131	.0114	.0100	.0087	.0077
4.	.0579	.0503	.0439	.0385	.0339	.0299
6.	.1205	.1056	.0929	.0820	.0726	.0644
8.	.1941	.1719	.1525	.1357	.1209	.1079
10.	.2705	.2422	.2170	.1946	.1747	.1570
12.	.3440	.3113	.2816	.2547	.2304	.2085
14.	.4113	.3756	.3430	.3129	.2852	.2598
16.	.4710	.4341	.3995	.3671	.3370	.3090
18.	.5228	.4855	.4501	.4165	.3847	.3549
20.	.5671	.5302	.4947	.4605	.4279	.3968
22.	.6047	.5686	.5334	.4992	.4662	.4345
24.	.6365	.6014	.5668	.5329	.4999	.4679
26.	.6632	.6291	.5953	.5619	.5292	.4972
28.	.6854	.6525	.6195	.5868	.5544	.5225
30.	.7039	.6720	.6399	.6077	.5758	.5442
32.	.7192	.6882	.6568	.6253	.5938	.5626
34.	.7316	.7014	.6707	.6398	.6088	.5778
36.	.7414	.7119	.6818	.6514	.6208	.5902
38.	.7490	.7200	.6904	.6604	.6302	.5998
40.	.7546	.7260	.6968	.6671	.6371	.6069
42.	.7582	.7299	.7009	.6714	.6416	.6116
44.	.7600	.7318	.7029	.6736	.6438	.6139

Table 8

Confidence Intervals for S_{xy}

k	$\hat{\theta}$	coh	$(\hat{S}_{xy})^{-1}$	Confidence interval on $1.0/S_{xy}(f)$	Width
20	-9.1	.13	-.954	(-3.34, 1.18)	4.52
16	-45.0	.61	-.299	(-.66, -.03)	.63
18	-15.6	.29	-.115	(-.32, .05)	.37
24	6.9	.08	.017	(-.028, .066)	.094
34	12.0	.21	.021	(-.007, .052)	.059
52	5.9	.06	.074	(-.084, .240)	.325
84	4.5	.03	.175	(-.249, .611)	.86
172	-2.6	.01	-.315	(-1.26, .62)	1.88

Note: Angles in degrees. For θ = 20 degrees, $1 - \alpha$ = .95.

PART 6: SUMMARY

A method for computing confidence intervals for the spectral energy estimates found in the monthly reports of the Coastal Data Information Program was presented. The chi-square values presented in Table 1 enable the quick computation of confidence intervals for any location and time given in the reports.

The probability law for the estimate of the longshore component of radiation stress denoted by $S_{xy}(f)$ was found to depend on the coherence of the longshore component with the offshore slope component. Since estimates of the coherence are not available, either finding a confidence interval that depends on the distribution of the spectral density estimates rather than the distribution of $S_{xy}(f)$ or obtaining values of the coherence by assuming a parameterized directional spectrum must be considered. The first approach is made possible by assuming a value of the apparent angle and then computing the interval of Equation (4.27). The second approach requires the assumption of a wrapped normal spreading function (or perhaps some other parameterized directional spectrum) for which the coherence is derived as a function and then computed for specific values of the angle parameter and the spread parameter (of Equation (5.22)). After the coherence is computed, the values of t and t are obtained using the program CIAB2 listed in Appendix D. Contained in Appendix D are listings of t and t for various values of the degrees of freedom k, confidence coefficient $1-$, and coherence COH. The first method provides intervals on the quantity $S_{xy}(f)$ itself. The second method has difficulties with cases where the constant t is less than zero and the constant t is greater. When this occurs, the intervals become infinite

in width and are therefore difficult to interpret. However, the confidence interval for the reciprocal of $S_{xy}(f)$ does not exhibit these difficulties. It is, therefore, presented as a much more easily understood interval. The interval for percent energy in Equation (2.12) has relative width

$$RW = k(1/\chi^2_{k,\alpha/2} - 1/\chi^2_{k,1-\alpha/2})$$

The relative width has values varying from .36 for period band 6-4 to 1.4 for period band 18-16. RW can be thought of as the resolution of the estimate with 90 percent certainty. Thus the percent energy can be estimated with 36 percent to 140 percent resolution depending upon the period band or value of k. Accordingly, the interval for radiation stress has relative width RW from Equation (5.13). RW is listed under W in the table of Appendix C. For typical values of the coherence from Table 8, RW varies from 2.11 to 5.90 resulting in a resolution of 211 percent to 590 percent with 95 percent certainty on the estimate of $1/S_{xy}(f)$. In all cases, the confidence intervals on $1/S_{xy}(f)$ contained the value zero; consequently, inferences about $S_{xy}(f)$ are difficult. Due to the apparent lack of resolution and to the difficulty in interpreting the intervals on $1/S_{xy}(f)$, it is concluded that the method given by Table 6 is the better of the two for general purposes.

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APPENDIX A

PROOF OF CONDITIONS FOR USING RESIDUE THEOREM

Case 1: Given $z > 0$ and the curve is chosen to be $t = Re^{-i\theta}$ $0 \le \theta \le \pi$.

In order to apply the residue theorem it must be true that

$$|Q_z(t)| \le M/|t|$$

for $|t|$ large, and no poles of $Q_z(t)$ on the real axis (Marsden, 1973).

Then $|t| = R$ and

$$Q_z(t) = (t-ia)^{-\gamma}(t-ib)^{-\gamma}$$

has poles on the real axis only if a=0 or b=0. This cannot happen due to the way in which a and b are defined. Now consider

$$|t-ia||t-ib| = |Re^{-i\theta}-ia||Re^{-i\theta}-ib| \ge \left||Re^{-i\theta}| - |ia|\right|\left||Re^{-i\theta}| - |ib|\right|$$

$$= |R-|a||\,|R-|b|| = |R^2-R(|a|+|b|) + |ab||$$

then

$$|t-ia||t-ib| \ge |R^2-R(|a|+|b|) + |ab||$$

and if R is large, then $R > |a| + |b|$ so

$$|t-ia||t-ib| > |R^2-R(|a| + |b|)|$$

Since

$$|t-ia||t-ib| > |R(R-(|a| + |b|))|$$

then

$$|t-ia||t-ib| > R(R-(|a| + |b|))$$

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[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.4216	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.3856	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.3516	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.3196	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.2893	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.2605	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.2332	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	1.[illegible]	1.[illegible]	.[illegible]	[illegible]	-.2073	[illegible]	[illegible]

CONFIDENCE MULTIPLIERS FOR
K = 16.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .900

CON	Z	T1	T2	T
.0100	9.0499	-7.0724	9.3383	16.3857
.0200	7.0000	-4.6616	6.8723	11.5343
.0300	5.6662	-3.5911	5.0613	8.6524
.0400	4.3990	-2.9511	3.1610	6.1121
.0500	4.3169	-2.5132	4.7228	7.2360
.0600	3.9561	-2.1891	4.3984	6.5875
.0700	3.6450	-1.9366	4.1416	6.0821
.0800	3.3912	-1.7325	3.9411	5.6737
.0900	3.1798	-1.5630	3.7713	5.3344
.1000	2.9000	-1.4193	3.6272	5.0465
.1100	2.8445	-1.2953	3.5029	4.7982
.1200	2.7080	-1.1869	3.3941	4.5611
.1300	2.5869	-1.0910	3.2979	4.3889
.1400	2.4765	-1.0054	3.2119	4.2173
.1500	2.3805	-.9283	3.1344	4.0626
.1600	2.2913	-.8583	3.0640	3.9224
.1700	2.2096	-.7945	2.9998	3.7943
.1800	2.1344	-.7359	2.9408	3.6767
.1900	2.0647	-.6819	2.8864	3.5682
.2000	2.0000	-.6318	2.8359	3.4677
.2100	1.9396	-.5852	2.7889	3.3742
.2200	1.8829	-.5418	2.7451	3.2869
.2300	1.8297	-.5011	2.7040	3.2051
.2400	1.7795	-.4629	2.6653	3.1282
.2500	1.7321	-.4269	2.6289	3.0558
.2600	1.6871	-.3929	2.5945	2.9874
.2700	1.6443	-.3608	2.5619	2.9227
.2800	1.6036	-.3303	2.5309	2.8612
.2900	1.5647	-.3013	2.5015	2.8029
.3000	1.5275	-.2738	2.4735	2.7472

CONFIDENCE MULTIPLIERS FOR
K = 16.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .900

CON	Z	T1	T2	T
.3100	1.4919	-.2475	2.4467	2.6942
.3200	1.4577	-.2224	2.4211	2.6435
.3300	1.4249	-.1984	2.3960	2.5950
.3400	1.3933	-.1754	2.3711	2.5465
.3500	1.3628	-.1533	2.3506	2.5038
.3600	1.3333	-.1321	2.3289	2.4610
.3700	1.3049	-.1117	2.3066	2.4197
.3800	1.2773	-.0921	2.2878	2.3800
.3900	1.2506	-.0732	2.2684	2.3416
.4000	1.2247	-.0550	2.2496	2.3046
.4100	1.1996	-.0374	2.2315	2.2689
.4200	1.1751	-.0204	2.2139	2.2343
.4300	1.1513	-.0040	2.1969	2.2009
.4400	1.1282	.0119	2.1804	2.1685
.4500	1.1055	.0273	2.1644	2.1371
.4600	1.0835	.0423	2.1489	2.1066
.4700	1.0619	.0568	2.1338	2.0770
.4800	1.0408	.0709	2.1191	2.0482
.4900	1.0202	.0845	2.1048	2.0203
.5000	1.0000	.0978	2.0909	1.9931
.5100	.9802	.1107	2.0773	1.9666
.5200	.9608	.1233	2.0641	1.9408
.5300	.9417	.1356	2.0512	1.9156
.5400	.9230	.1475	2.0386	1.8911
.5500	.9045	.1591	2.0263	1.8672
.5600	.8864	.1705	2.0142	1.8438
.5700	.8686	.1815	2.0025	1.8210
.5800	.8510	.1923	1.9910	1.7987
.5900	.8336	.2029	1.9797	1.7768
.6000	.8165	.2132	1.9687	1.7555

CONFIDENCE MULTIPLIERS FOR
N = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .800

COH	D	T1	T2	F
.3100	1.4810	-.1200	2.2666	2.4134
.3200	1.4577	-.1044	2.2637	2.3651
.3300	1.4246	-.0829	2.2410	2.3247
.3400	1.3753	-.0623	2.2209	2.2831
.3500	1.3628	-.0426	2.2007	2.2432
.3600	1.3333	-.0236	2.1813	2.2049
.3700	1.3049	-.0054	2.1626	2.1680
.3800	1.2773	.0122	2.1446	2.1324
.3900	1.2506	.0291	2.1273	2.0982
.4000	1.2247	.0454	2.1105	2.0651
.4100	1.1996	.0612	2.0943	2.0332
.4200	1.1751	.0764	2.0786	2.0023
.4300	1.1513	.0911	2.0635	1.9723
.4400	1.1282	.1054	2.0487	1.9434
.4500	1.1055	.1192	2.0345	1.9153
.4600	1.0835	.1326	2.0206	1.8880
.4700	1.0619	.1455	2.0071	1.8616
.4800	1.0408	.1582	1.9940	1.8358
.4900	1.0202	.1704	1.9812	1.8108
.5000	1.0000	.1823	1.9688	1.7865
.5100	.9802	.1939	1.9567	1.7628
.5200	.9608	.2052	1.9449	1.7397
.5300	.9417	.2162	1.9334	1.7172
.5400	.9230	.2269	1.9222	1.6953
.5500	.9045	.2373	1.9112	1.6739
.5600	.8864	.2475	1.9004	1.6530
.5700	.8686	.2574	1.8900	1.6325
.5800	.8510	.2671	1.8797	1.6126
.5900	.8336	.2766	1.8696	1.5931
.6000	.8165	.2858	1.8598	1.5740

CONFIDENCE MULTIPLIERS FOR
N = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .900

COH	D	T1	T2	F
.6100	.7996	.2949	1.8502	1.5553
.6200	.7829	.3037	1.8407	1.5370
.6300	.7664	.3124	1.8314	1.5191
.6400	.7500	.3208	1.8224	1.5015
.6500	.7336	.3291	1.8134	1.4843
.6600	.7177	.3372	1.8047	1.4674
.6700	.7018	.3452	1.7961	1.4509
.6800	.6860	.3530	1.7876	1.4346
.6900	.6703	.3606	1.7795	1.4187
.7000	.6547	.3681	1.7711	1.4030
.7100	.6391	.3755	1.7631	1.3876
.7200	.6236	.3827	1.7552	1.3725
.7300	.6082	.3898	1.7474	1.3576
.7400	.5927	.3967	1.7397	1.3430
.7500	.5774	.4036	1.7322	1.3286
.7600	.5620	.4103	1.7247	1.3144
.7700	.5465	.4169	1.7174	1.3005
.7800	.5311	.4234	1.7101	1.2868
.7900	.5156	.4297	1.7030	1.2733
.8000	.5000	.4360	1.6960	1.2600
.8100	.4843	.4421	1.6890	1.2469
.8200	.4685	.4482	1.6821	1.2339
.8300	.4526	.4541	1.6754	1.2212
.8400	.4364	.4600	1.6687	1.2087
.8500	.4201	.4658	1.6620	1.1963
.8600	.4035	.4714	1.6555	1.1840
.8700	.3866	.4770	1.6490	1.1720
.8800	.3693	.4825	1.6426	1.1601
.8900	.3516	.4880	1.6363	1.1483
.9000	.3333	.4933	1.6300	1.1367

CONFIDENCE MULTIPLIERS FOR
N = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .600

[illegible]	[illegible]	T1	T2	W
[illegible]	[illegible]	.6606	1.3611	.7307
[illegible]	[illegible]	.6547	1.3766	.7221
[illegible]	.7664	.6568	1.3726	.7136
[illegible]	.7603	.6622	1.3684	.7056
[illegible]	.7536	.6666	1.3643	.6975
[illegible]	.7477	.6707	1.3603	.6896
[illegible]	.7416	.6745	1.3564	.6819
[illegible]	.6860	.6782	1.3525	.6743
[illegible]	.6773	.6818	1.3487	.6662
[illegible]	.6647	.6854	1.3450	.6595
[illegible]	.6591	.6890	1.3413	.6523
[illegible]	.6536	.6924	1.3377	.6452
[illegible]	[illegible]	.6958	1.3341	.6383
[illegible]	.5927	.6992	1.3306	.6315
[illegible]	.5774	.7025	1.3272	.6247
[illegible]	.5620	.7057	1.3238	.6181
[illegible]	.5466	.7089	1.3204	.6116
[illegible]	.5311	.7120	1.3172	.6052
[illegible]	.5156	.7151	1.3139	.5988
[illegible]	.5000	.7181	1.3107	.5926
[illegible]	.4843	.7211	1.3076	.5865
[illegible]	.4685	.7240	1.3044	.5804
[illegible]	.4526	.7269	1.3014	.5745
[illegible]	.4364	.7298	1.2983	.5686
[illegible]	.4201	.7326	1.2953	.5628
[illegible]	.4035	.7353	1.2924	.5571
[illegible]	.3866	.7381	1.2895	.5514
[illegible]	.3693	.7408	1.2866	.5458
[illegible]	.3516	.7434	1.2837	.5403
[illegible]	.3333	.7460	1.2809	.5349

CONFIDENCE MULTIPLIERS FOR
N = 20.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .900

COH	D	T1	T2	W
.0100	9.9499	-6.2328	8.4024	14.6353
.0200	7.0000	-4.0760	6.2453	10.3214
.0300	5.6662	-3.1180	5.2070	8.4050
.0400	4.8990	-2.5454	4.7141	7.2595
.0500	4.3589	-2.1537	4.3221	6.4758
.0600	3.9581	-1.8637	4.0319	5.8956
.0700	3.6450	-1.6378	3.8057	5.4435
.0800	3.3912	-1.4553	3.6228	5.0781
.0900	3.1798	-1.3037	3.4709	4.7746
.1000	3.0000	-1.1751	3.3421	4.5171
.1100	2.8445	-1.0642	3.2309	4.2951
.1200	2.7080	-.9672	3.1336	4.1008
.1300	2.5869	-.8814	3.0475	3.9289
.1400	2.4785	-.8048	2.9706	3.7754
.1500	2.3805	-.7358	2.9013	3.6371
.1600	2.2913	-.6733	2.8384	3.5117
.1700	2.2096	-.6162	2.7810	3.3971
.1800	2.1344	-.5637	2.7282	3.2920
.1900	2.0647	-.5154	2.6795	3.1950
.2000	2.0000	-.4706	2.6344	3.1051
.2100	1.9396	-.4290	2.5924	3.0214
.2200	1.8829	-.3901	2.5532	2.9433
.2300	1.8297	-.3537	2.5165	2.8702
.2400	1.7795	-.3195	2.4819	2.8015
.2500	1.7321	-.2873	2.4494	2.7367
.2600	1.6871	-.2570	2.4186	2.6756
.2700	1.6443	-.2282	2.3895	2.6177
.2800	1.6036	-.2009	2.3618	2.5628
.2900	1.5647	-.1750	2.3355	2.5106
.3000	1.5275	-.1504	2.3105	2.4608

CONFIDENCE MULTIPLIERS FOR
K = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .900

SCP	D	T1	T2	W
.0700	9.9499	-2.4000	4.4371	6.8371
.0800	7.0000	-1.3926	3.4277	4.8203
.0900	5.6662	-.9452	2.9822	3.9274
.0400	4.8990	-.6776	2.7147	3.3923
.0900	4.3166	-.4948	2.5317	3.0265
.0800	3.8581	-.3595	2.3961	2.7557
.0700	3.6450	-.2540	2.2906	2.5446
.0600	3.3912	-.1667	2.2053	2.3720
.0500	3.1798	-.0979	2.1345	2.2324
.1000	3.0000	-.0379	2.0743	2.1122
.1100	2.8445	.0139	2.0225	2.0086
.1200	2.7080	.0592	1.9771	1.9179
.1300	2.5869	.0993	1.9370	1.8377
.1400	2.4785	.1350	1.9011	1.7661
.1500	2.3805	.1673	1.8688	1.7016
.1600	2.2913	.1965	1.8395	1.6431
.1700	2.2096	.2232	1.8128	1.5896
.1800	2.1344	.2477	1.7882	1.5406
.1900	2.0647	.2702	1.7656	1.4953
.2000	2.0000	.2912	1.7446	1.4534
.2100	1.9396	.3106	1.7250	1.4144
.2200	1.8829	.3288	1.7066	1.3780
.2300	1.8297	.3458	1.6897	1.3439
.2400	1.7795	.3618	1.6736	1.3118
.2500	1.7321	.3768	1.6585	1.2816
.2600	1.6871	.3911	1.6442	1.2531
.2700	1.6443	.4045	1.6307	1.2261
.2800	1.6036	.4173	1.6178	1.2005
.2900	1.5647	.4294	1.6056	1.1762
.3000	1.5275	.4409	1.5940	1.1530

CONFIDENCE MULTIPLIERS FOR
K = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .900

SCP	D	T1	T2	W
.3100	1.4919	.4520	1.5329	1.1809
.3200	1.4577	.4625	1.5722	1.1098
.3300	1.4249	.4726	1.5621	1.0895
.3400	1.3933	.4822	1.5524	1.0702
.3500	1.3628	.4915	1.5430	1.0515
.3600	1.3333	.5003	1.5340	1.0337
.3700	1.3049	.5089	1.5254	1.0165
.3800	1.2773	.5171	1.5170	.9999
.3900	1.2506	.5251	1.5090	.9839
.4000	1.2247	.5327	1.5012	.9685
.4100	1.1996	.5401	1.4937	.9536
.4200	1.1751	.5473	1.4865	.9392
.4300	1.1513	.5542	1.4795	.9253
.4400	1.1282	.5609	1.4727	.9117
.4500	1.1055	.5674	1.4661	.8986
.4600	1.0835	.5737	1.4596	.8859
.4700	1.0619	.5798	1.4534	.8736
.4800	1.0408	.5858	1.4474	.8616
.4900	1.0202	.5916	1.4415	.8499
.5000	1.0000	.5972	1.4358	.8386
.5100	.9802	.6027	1.4302	.8275
.5200	.9608	.6080	1.4247	.8168
.5300	.9417	.6132	1.4194	.8063
.5400	.9230	.6182	1.4142	.7960
.5500	.9045	.6232	1.4092	.7860
.5600	.8864	.6280	1.4043	.7763
.5700	.8686	.6327	1.3994	.7667
.5800	.8510	.6373	1.3947	.7574
.5900	.8336	.6418	1.3901	.7483
.6000	.8165	.6462	1.3856	.7394

Note: k = twice the number of raw spectral lines in estimate averaging 2. from Equation (4.2); confidence coefficient = 1-α from Equation (4.7); D = parameter from Equation (4.3); T1 and T2 = t_{α} and t_{β} from Equation (5.10); and W = relative width (t_{β} - t_{α}).

APPENDIX C

TABLES OF CONFIDENCE MULTIPLIERS FOR $S_{xy}(f)$ CONFIDENCE INTERVALS GIVEN A SPECIFIED COHERENCE

CRESCENT CITY ARRAY, DIRECTION
MAY 1981

PST DAY/TIME		SIG. ANG (DEG)	TOT SXY (CM SQ)	ANGULAR DISTRIBUTION IN PERIOD BANDS (ANGLES IN DEGREES) BAND PERIOD LIMITS (SECS) 22+	22-18	18-16	16-14	14-12	12-10	10-8	8-6	6-4
1	0320	59.8	-6.3	30.1	15.0	36.6	43.7	60.6	66.0	66.2	62.1
1	0926	63.1	18.6	39.3	38.6	38.9	51.8	70.8	66.8	67.8	67.3
1	1521	61.7	13.6	32.1	36.6	48.7	59.5	66.0	65.4	64.2	59.0
1	2120	65.9	99.5	50.9	15.0	44.4	66.9	72.0	65.9	64.5	57.4
2	0322	64.6	58.2	48.9	44.0	56.9	57.1	70.1	66.6	66.1	62.3
2	0920	64.7	38.4	33.6	31.6	53.9	64.5	65.1	66.2	66.5	63.1
2	1520	59.5	-7.9	28.7	33.6	32.7	55.7	58.7	64.1	62.5	60.3
2	2119	67.4	50.9	43.5	40.0	35.4	69.7	68.2	70.6	69.4	64.5
3	0321	62.2	15.6	52.1	36.6	23.8	56.6	64.9	67.6	60.4	57.1
3	0920	65.7	24.0	46.8	19.8	47.0	42.2	69.0	67.1	70.9	63.4
3	1519	62.0	5.9	28.1	29.3	39.1	60.0	67.0	66.4	64.3	58.5
3	2120	59.6	-5.0	42.0	44.5	36.7	36.4	60.9	67.7	66.3	64.1
4	0320	62.1	10.1	44.4	30.3	38.4	52.8	59.2	66.2	63.1	59.0
4	0922	64.3	15.5	47.5	35.9	39.0	44.8	63.0	69.5	69.8	64.4
4	1524	60.7	-0.0	39.5	32.9	29.2	33.2	66.9	65.2	64.4	60.9
4	2123	60.9	-4.2	46.1	28.5	36.0	43.1	64.3	67.0	68.7	64.4
5	0321	63.6	11.3	36.7	42.1	15.0	51.9	69.2	67.3	65.9	60.7
5	1007	64.6	10.9	39.6	40.0	39.8	42.7	66.1	72.7	67.0	69.4
5	1523	60.9	-2.0	32.0	32.3	36.2	36.0	67.6	63.9	65.7	63.4
5	2120	61.3	-0.6	38.5	24.2	38.0	44.4	69.6	67.6	69.2	64.3
6	0320	59.3	-8.4	15.0	32.8	15.0	40.0	61.6	66.7	64.6	62.2
6	0925	59.7	-17.7	37.3	37.9	33.5	41.7	51.8	68.7	66.0	55.5
6	1524	55.8	-27.6	33.3	30.4	39.6	33.1	60.1	66.0	63.2	66.2
6	2121	56.8	-32.6	34.0	39.9	15.0	35.4	54.6	62.8	69.6	68.0
7	0320	56.3	-22.9	36.8	37.0	37.1	42.6	53.1	72.0	64.0	66.2
7	0923	49.6	-60.4	15.0	38.0	21.9	30.6	56.4	62.7	64.3	59.9
7	1522	49.9	-23.5	46.0	44.1	33.4	35.4	54.0	69.3	68.9	61.8
8	2122	51.1	-75.4	15.0	15.0	26.5	40.9	55.4	52.9	65.5	66.5
9	0320	51.6	-56.2	35.5	24.7	38.8	45.5	62.5	61.4	65.1	72.3
9	0919	52.1	-37.7	41.8	29.0	31.1	20.3	56.5	62.5	67.6	61.7
9	1520	63.6	3.9	26.2	32.7	39.3	41.8	68.9	70.3	69.3	65.0
9	2122	63.1	2.8	34.9	35.0	34.3	44.9	61.7	68.9	69.1	62.1
10	0320	64.9	14.7	43.4	31.9	48.9	23.9	64.9	67.2	71.2	68.0

CRESCENT CITY ARRAY, ENERGY
MAY 1981

LOCAL DAY/TIME	SIG HT (CM)	TOT EN (CM SQ)	PERCENT ENERGY IN BAND (TOTAL ENERGY INCLUDES RANGE 2048-4 SECS) BAND PERIOD LIMITS (SECS) 22+	22-18	18-16	16-14	14-12	12-10	10-8	8-6	6-4
1 0320	75.4	355.0	1.5	1.7	3.6	8.0	4.5	20.0	24.9	21.8	14.4
1 0926	91.1	518.3	1.5	0.4	3.7	9.4	8.2	20.5	32.5	13.6	10.7
1 1521	105.7	698.1	2.2	1.3	1.6	9.4	18.1	28.0	20.1	10.9	8.7
1 2120	133.9	1120.8	3.0	0.6	0.6	3.0	43.8	20.7	12.3	7.4	9.1
2 0322	118.6	879.7	3.4	0.5	0.8	2.3	20.0	30.3	21.7	10.1	11.4
2 0920	100.5	630.7	2.9	0.4	1.1	3.3	10.1	21.2	32.0	17.9	11.6
2 1520	102.9	661.3	1.9	0.5	1.5	3.3	10.7	30.9	25.7	14.5	11.6
2 2119	92.2	531.2	4.6	0.3	1.6	2.8	5.9	33.4	27.1	14.4	10.4
3 0321	88.5	489.3	4.3	0.3	2.5	1.8	2.9	31.8	25.6	17.9	13.3
3 0920	75.5	356.2	4.3	1.8	1.6	2.4	2.7	7.7	42.6	23.0	14.2
3 1519	75.3	354.6	7.6	2.0	5.2	2.9	1.9	11.2	35.7	19.4	14.5
3 2120	62.4	243.0	7.2	1.5	16.1	3.6	4.6	11.3	26.0	15.9	14.3
4 0320	81.6	416.2	9.3	0.8	3.0	2.7	2.3	4.0	43.7	24.6	10.1
4 0922	72.5	328.4	9.1	1.0	4.3	4.4	3.0	7.9	31.6	21.5	17.5
4 1524	75.1	352.0	6.3	1.3	4.7	4.2	3.8	6.7	42.8	16.4	14.3
4 2123	70.6	311.8	8.7	6.8	5.7	6.7	4.1	9.9	35.3	10.9	12.2
5 0321	71.8	322.1	12.0	2.3	2.7	2.8	6.4	20.4	29.4	13.6	10.9
5 1007	69.0	297.5	11.0	3.8	3.7	8.5	2.3	17.1	25.8	19.3	8.9
5 1523	61.7	238.0	9.3	3.3	3.5	6.8	3.0	14.5	30.5	15.5	13.9
5 2120	60.1	225.5	8.8	3.1	8.1	6.6	4.8	13.3	27.2	14.0	14.5
6 0320	59.7	222.8	14.6	4.5	5.4	7.6	2.7	7.4	35.4	15.5	7.2
6 0925	61.6	237.0	6.9	20.0	8.3	5.3	2.8	4.0	35.7	9.0	8.6
6 1524	57.3	205.4	8.1	22.1	10.2	9.2	3.8	5.1	20.9	12.4	8.7
6 2121	58.0	210.0	5.3	30.4	10.5	6.2	3.7	3.6	16.4	10.7	13.7
7 0320	54.0	182.5	13.1	24.0	7.9	8.6	7.5	4.6	11.0	12.8	10.9
7 0923	64.0	255.9	4.2	21.7	29.2	4.9	4.0	2.3	17.6	7.5	8.9
7 1522	47.5	141.2	2.2	10.6	37.1	16.8	4.4	4.1	10.0	8.4	6.8
7 2121	53.1	176.1	4.2	13.8	37.8	10.6	7.3	7.3	11.6	4.7	3.2
8 0320	51.7	167.2	10.0	10.0	24.7	18.7	6.2	7.3	11.5	6.1	6.0
8 0922	74.2	344.4	1.8	5.1	49.3	17.5	2.6	1.1	16.9	3.6	2.7
8 1520	51.9	168.6	4.3	6.3	24.8	35.9	9.6	5.7	5.5	5.8	2.5
8 2122	81.3	412.8	1.4	9.2	15.7	14.6	3.6	2.1	10.6	25.5	17.8
9 0320	78.0	380.5	2.2	3.2	10.1	27.6	13.6	2.4	17.3	15.3	8.7

APPENDIX B

PAGES FROM ORIGINAL REPORT FOR WHICH EXAMPLE CONFIDENCE INTERVALS ARE COMPUTED

and if R is large enough, then $R-(|a|+|b|) \geq 1.0$ and thus for sufficiently large $|t| = R$

$$|t-ia||t-ib| \geq R$$

Therefore

$$|Q_z(t)| = \{|t-ia||t-ib|\}^{-\nu} \leq R^{-\nu}$$

and then

$$|Q_z(t)| \leq 1/R^{\nu}$$

but

$$1/R^{\nu} \leq 1/R = 1/|t|$$

The boundedness condition is therefore satisfied by M = 1.0.

Case 2: Given $z < 0$ and the curve Γ is chosen to be $t = Re^{i\theta}$ $0 \leq \theta \leq \pi$. Then

$$|t-ia||t-ib| = |Re^{i\theta}-ia||Re^{i\theta}-ib| \geq |R-|a|||R-|b||$$

and the result will be identical to that of Case 1.

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CONFIDENCE COEFFICIENT = .90

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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.1300	[illegible]	-.1892	[illegible]	[illegible]
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.8000	.5000	[illegible]	[illegible]	[illegible]
.8100	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.7994	[illegible]	[illegible]
[illegible]	[illegible]	.8014	[illegible]	[illegible]
.8600	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.8055	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.8094	[illegible]	[illegible]
.9000	[illegible]	.8114	[illegible]	[illegible]

CONFIDENCE MULTIPLIERS FOR
N = 92.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .950

CCH	T	T1	T2	W
.0100	9.9499	[illegible]	[illegible]	[illegible]
.0200	7.0000	-1.8411	[illegible]	[illegible]
.0300	5.6862	-1.3167	[illegible]	[illegible]
.0400	4.8990	[illegible]	[illegible]	[illegible]
.0500	4.3589	[illegible]	[illegible]	[illegible]
.0600	3.9581	[illegible]	[illegible]	[illegible]
.0700	3.6450	[illegible]	[illegible]	3.0407
.0800	3.3912	[illegible]	[illegible]	[illegible]
.0900	3.1798	[illegible]	[illegible]	[illegible]
.1000	3.0000	[illegible]	[illegible]	2.5236
.1100	2.8445	[illegible]	[illegible]	2.3999
.1200	2.7080	[illegible]	[illegible]	2.2915
.1300	2.5869	[illegible]	[illegible]	2.1957
.1400	2.4785	-.0251	[illegible]	2.1100
.1500	2.3805	.0134	[illegible]	2.0329
.1600	2.2913	.0484	[illegible]	[illegible]
.1700	2.2096	[illegible]	[illegible]	[illegible]
.1800	2.1344	.1095	1.9499	[illegible]
.1900	2.0647	.1365	[illegible]	[illegible]
.2000	2.0000	.1615	[illegible]	1.7361
.2100	1.9396	.1848	[illegible]	[illegible]
.2200	1.8829	.2065	[illegible]	[illegible]
.2300	1.8297	.2266	1.8320	1.6051
.2400	1.7795	.2459	1.8127	[illegible]
.2500	1.7321	.2639	1.7946	1.5307
.2600	1.6871	.2809	1.7775	[illegible]
.2700	1.6443	.2970	1.7613	[illegible]
.2800	1.6036	.3122	1.7459	[illegible]
.2900	1.5647	.3267	1.7313	1.4046
.3000	1.5275	.3405	1.7174	[illegible]

CONFIDENCE MULTIPLIERS FOR
R = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COR	T	T1	T2	T
[illegible]	1.4914	.3837	[illegible]	[illegible]
[illegible]	1.4577	[illegible]	1.6314	1.[illegible]
[illegible]	1.4249	.3782	1.6796	1.[illegible]
[illegible]	1.4923	.3698	1.6675	1.[illegible]
[illegible]	1.4628	.4008	1.[illegible]	1.[illegible]
.4600	1.3333	.4114	1.[illegible]	1.0341
.4700	1.3049	.4216	1.6352	1.0136
.4800	1.2773	.4315	1.6251	1.1[illegible]
.4900	1.2506	.4410	1.6156	1.1746
.5000	1.2247	.4501	1.6066	1.1662
.5100	1.1996	.4589	1.5973	1.1584
.5200	1.1751	.4675	1.5886	1.1511
.5300	1.1513	.4757	1.5802	1.1045
.5400	1.1281	.4837	1.5720	1.0883
.5500	1.1055	.4915	1.5641	1.0736
.5600	1.0835	.4990	1.5564	1.0574
.5700	1.0619	.5063	1.5489	1.0427
.5800	1.0408	.5134	1.5417	1.0283
.5900	1.0202	.5203	1.5346	1.0144
.6000	1.0000	.5270	1.5277	1.0008
.6100	.9802	.5335	1.5210	.9876
.6200	.9608	.5398	1.5145	.9747
.6300	.9417	.5460	1.5081	.9621
.6400	.9230	.5520	1.5019	.9499
.6500	.9045	.5579	1.4959	.9379
.6600	.8864	.5637	1.4899	.9263
.6700	.8686	.5693	1.4841	.9149
.6800	.8510	.5747	1.4785	.9037
.6900	.8336	.5801	1.4729	.8928
.7000	.8165	.5853	1.4675	.8822

CONFIDENCE MULTIPLIERS FOR
R = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .950

COR	T	T1	T2	T
.6100	.7996	.5904	1.4[illegible]	.6717
.6200	.7829	.5954	1.4[illegible]	.6[illegible]
.6300	.7664	.6003	1.4516	.6615
.6400	.7500	.6051	1.4465	.6517
.6500	.7338	.6098	1.4414	.6421
.6600	.7177	.6144	1.4371	.6327
.6700	.7018	.6189	1.4323	.6234
.6800	.6860	.6235	1.4277	.6044
.6900	.6703	.6277	1.4231	.7954
.7000	.6547	.6319	1.4186	.7867
.7100	.6391	.6361	1.4142	.7781
.7200	.6236	.6402	1.4099	.7696
.7300	.6082	.6442	1.4056	.7613
.7400	.5927	.6482	1.4014	.7532
.7500	.5774	.6521	1.3972	.7451
.7600	.5620	.6559	1.3931	.7372
.7700	.5465	.6597	1.3891	.7294
.7800	.5311	.6634	1.3851	.7217
.7900	.5156	.6670	1.3812	.7142
.8000	.5000	.6706	1.3774	.7068
.8100	.4843	.6741	1.3735	.6994
.8200	.4685	.6776	1.3696	.6922
.8300	.4526	.6810	1.3661	.6851
.8400	.4364	.6844	1.3624	.6781
.8500	.4201	.6877	1.3588	.6711
.8600	.4035	.6909	1.3552	.6643
.8700	.3866	.6941	1.3517	.6575
.8800	.3693	.6973	1.3482	.6509
.8900	.3516	.7004	1.3447	.6443
.9000	.3333	.7035	1.3413	.6378

[illegible] MULTIPLIERS FOR
[illegible] DEGREES OF FREEDOM
[illegible] COEFFICIENT = .950

[illegible]	[illegible]	T1	T2	[illegible]
[illegible]	[illegible]	-7.7006	9.9734	17.6740
[illegible]	[illegible]	[illegible]	7.3676	12.4631
[illegible]	[illegible]	[illegible]	6.2101	10.1462
[illegible]	[illegible]	[illegible]	[illegible]	8.7641
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	-2.4225	4.6935	7.1160
[illegible]	[illegible]	-2.1495	4.4201	6.5696
[illegible]	[illegible]	-1.9288	4.1991	6.1279
[illegible]	[illegible]	-1.7456	4.0155	5.7611
[illegible]	[illegible]	-1.5902	[illegible]	5.4498
[illegible]	[illegible]	-1.4561	3.7252	5.1813
[illegible]	[illegible]	-1.3388	3.6076	4.9464
[illegible]	[illegible]	-1.2351	3.5035	4.7386
[illegible]	[illegible]	-1.1425	3.4104	4.5529
[illegible]	2.3805	-1.0590	3.3266	4.3856
.1600	2.2913	-.9834	3.2505	4.2339
.1700	2.2096	-.9143	3.1810	4.0953
.1800	2.1344	-.8509	3.1172	3.9661
.1900	2.0647	-.7924	3.0583	3.8507
.2000	2.0000	-.7383	3.0036	3.7419
.2100	1.9396	-.6879	2.9526	3.6407
.2200	1.8829	-.6408	2.9053	3.5461
.2300	1.8297	-.5968	2.8608	3.4576
.2400	1.7795	-.5554	2.8190	3.3744
.2500	1.7321	-.5165	2.7795	3.2960
.2600	1.6871	-.4797	2.7423	3.2220
.2700	1.6443	-.4449	2.7070	3.1519
.2800	1.6036	-.4119	2.6735	3.0854
.2900	1.5647	-.3805	2.6416	3.0221
.3000	1.5275	-.3507	2.6112	2.9619

CONFIDENCE MULTIPLIERS FOR
K = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .950

CON	S	T1	T2	[illegible]
.3100	1.4919	-.3222	[illegible]	[illegible]
.3200	1.4677	-.2950	[illegible]	[illegible]
.3300	1.4249	-.2690	[illegible]	[illegible]
.3400	1.3933	-.2441	[illegible]	[illegible]
.3500	1.3628	-.2202	[illegible]	[illegible]
.3600	1.3333	-.1972	[illegible]	[illegible]
.3700	1.3049	-.1752	[illegible]	[illegible]
.3800	1.2773	-.1540	[illegible]	[illegible]
.3900	1.2506	-.1335	[illegible]	[illegible]
.4000	1.2247	-.1136	[illegible]	[illegible]
.4100	1.1996	-.0947	2.3490	[illegible]
.4200	1.1751	-.0763	2.3299	[illegible]
.4300	1.1513	-.0585	2.3115	[illegible]
.4400	1.1282	-.0413	[illegible]	[illegible]
.4500	1.1055	-.0246	[illegible]	[illegible]
.4600	1.0835	-.0084	[illegible]	[illegible]
.4700	1.0619	.0073	[illegible]	[illegible]
.4800	1.0408	[illegible]	[illegible]	[illegible]
.4900	1.0202	[illegible]	[illegible]	[illegible]
.5000	1.0000	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.5400	.9836	[illegible]	[illegible]	[illegible]
.5500	.9645	[illegible]	[illegible]	[illegible]
.5600	.9464	[illegible]	[illegible]	[illegible]
.5700	.8286	[illegible]	[illegible]	[illegible]
.5800	.8510	[illegible]	[illegible]	[illegible]
.5900	.8336	[illegible]	[illegible]	[illegible]
.6000	.8165	[illegible]	[illegible]	[illegible]

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[illegible]
[illegible]
[illegible] COEFFICIENT = [illegible]
 CCR        I          T1         T2         [illegible]
 .6100   [illegible] [illegible] [illegible] [illegible]
 .6200   [illegible] [illegible] [illegible] 1.8424
 .6300   [illegible] [illegible] [illegible] [illegible]
 .6400   [illegible] [illegible] [illegible] 1.7[illegible]
 .6500   [illegible] [illegible] [illegible] [illegible]
 .6600   [illegible] [illegible] [illegible] 1.7562
 .6700   [illegible] [illegible] 1.9[illegible] 1.7[illegible]
 .6800   [illegible] [illegible] 1.9[illegible] 1.71[illegible]
 .6900   [illegible] [illegible] 1.965[illegible] 1.6992
 .7000   [illegible] [illegible] 1.9559   1.6802
 .7100    .6391   [illegible] 1.9455   1.6616
 .7200    .6236    .2924   1.9356   1.643[illegible]
 .7300   [illegible] [illegible] 1.9263   1.6254
 .7400    .5927    .3090   1.9169   1.6077
 .7500    .5774    .3173   1.9076   1.5903
 .7600    .5620    .3255   1.8985   1.5732
 .7700    .5465    .3331   1.8895   1.5564
 .7800    .5311    .3408   1.8806   1.5399
 .7900    .5156    .3483   1.8719   1.5236
 .8000    .5000    .3557   1.8632   1.5075
 .8100    .4843    .36[illegible] 1.8547   1.4917
 .8200    .4685    .3701   1.8463   1.4762
 .8300    .4526    .3771   1.8379   1.4608
 .8400    .4364    .3846   1.8297   1.4457
 .8500    .4201    .3907   1.8215   1.4308
 .8600    .4035    .3974   1.8135   1.4161
 .8700    .3866    .4039   1.8055   1.4016
 .8800    .3693    .4103   1.7976   1.3873
 .8900    .3516    .4167   1.7898   1.3732
 .9000    .3333    .4229   1.7821   1.3592
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CONFIDENCE MULTIPLIERS FOR
N = 1[illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]
 CCR        I          T1         T2         [illegible]
 .0100    9.9499   -6.7386   11.[illegible] [illegible]
 .0200    7.0000   -5.6171   [illegible] [illegible]
 .0300    5.6862   -4.5194   [illegible] 11.[illegible]
 .0400    4.8990   -3.7437   [illegible] [illegible]
 .0500    4.3589   -3.2129   [illegible] [illegible]
 .0600    3.9581   -2.8201   [illegible] [illegible]
 .0700    3.6450   -2.[illegible] [illegible] [illegible]
 .0800    3.3912   -2.2061   4.[illegible] [illegible]
 .0900    3.1798   -2.0609   [illegible] [illegible]
 .1000    3.0000   -1.8806   [illegible] [illegible]
 .1100    2.8445   -1.7362   4.[illegible] [illegible]
 .1200    2.7080   -1.6047   [illegible] [illegible]
 .1300    2.5869   -1.4884   [illegible] [illegible]
 .1400    2.4785   -1.3845   [illegible] [illegible]
 .1500    2.3805   -1.2909   3.[illegible] [illegible]
 .1600    2.2913   -1.2060   3.[illegible] 4.7[illegible]
 .1700    2.2096   -1.128[illegible] [illegible] [illegible]
 .1800    2.1344   -1.0577   [illegible] [illegible]
 .1900    2.0647    -.9917   [illegible] [illegible]
 .2000    2.0000    -.9[illegible] [illegible] [illegible]
 .2100    1.9396    -.8744   [illegible] [illegible]
 .2200    1.8829    -.8216   [illegible] [illegible]
 .2300    1.8297    -.77[illegible] [illegible] [illegible]
 .2400    1.7795    -.7[illegible] [illegible] [illegible]
 .2500    1.7321    -.6821   [illegible] [illegible]
 .2600    1.6871    -.6403   [illegible] [illegible]
 .2700    1.6443    -.6017    2.9167   [illegible]
 .2800    1.6036    -.5647    2.[illegible] [illegible]
 .2900    1.5647    -.5296   [illegible] [illegible]
 .3000    1.5275    -.4960    2.8191   [illegible]
```

[illegible] MULTIPLIERS FOR
[illegible] DEGREES OF FREEDOM
[illegible] COEFFICIENT = .90

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	1.4[illegible]	-.4[illegible]	2.[illegible]	3.[illegible]
[illegible]	1.4[illegible]	-.4[illegible]	2.[illegible]	3.1[illegible]
[illegible]	1.4[illegible]	-.4[illegible]	2.[illegible]	3.1[illegible]
[illegible]	1.[illegible]	-.3[illegible]	2.6[illegible]	3.0[illegible]
[illegible]	1.[illegible]	-.3[illegible]	2.6[illegible]	3.0[illegible]
[illegible]	1.3[illegible]	-.3[illegible]	2.6[illegible]	2.9[illegible]
[illegible]	1.3[illegible]	-.2[illegible]	2.5[illegible]	2.9[illegible]
[illegible]	1.2[illegible]	-.2[illegible]	2.5[illegible]	2.8[illegible]
[illegible]	1.2[illegible]	-.2[illegible]	2.5[illegible]	2.8[illegible]
[illegible]	1.2[illegible]	-.2[illegible]	2.5[illegible]	2.7[illegible]
[illegible]	1.1[illegible]	-.2[illegible]	2.5[illegible]	2.7[illegible]
[illegible]	1.1[illegible]	-.1[illegible]	2.5[illegible]	2.6[illegible]
[illegible]	1.1[illegible]	-.1[illegible]	2.4[illegible]	2.6[illegible]
[illegible]	1.1[illegible]	-.1[illegible]	2.4[illegible]	2.6[illegible]
[illegible]	1.1[illegible]	-.1[illegible]	2.4[illegible]	2.5[illegible]
[illegible]	1.0[illegible]	-.1[illegible]	2.4[illegible]	2.5[illegible]
[illegible]	1.0[illegible]	-.1[illegible]	2.4[illegible]	2.5[illegible]
[illegible]	1.0[illegible]	-.0[illegible]	2.3[illegible]	2.4[illegible]
[illegible]	1.0[illegible]	-.0[illegible]	2.3[illegible]	2.4[illegible]
[illegible]	1.0[illegible]	-.0[illegible]	2.3[illegible]	2.4[illegible]
[illegible]	1.0000	-.0444	2.3530	2.3974
[illegible]	.9[illegible]	-.0[illegible]	2.3364	2.3[illegible]
[illegible]	.9[illegible]	-.0[illegible]	2.3203	2.3338
[illegible]	.9417	.0014	2.3045	2.3032
[illegible]	.9[illegible]	.0158	2.2891	2.2733
[illegible]	.9[illegible]	.0299	2.2741	2.2442
[illegible]	.8[illegible]	.0436	2.2594	2.2157
[illegible]	.8[illegible]	.0570	2.2450	2.1880
[illegible]	.8510	.0701	2.2309	2.1608
[illegible]	.8[illegible]	.0829	2.2171	2.1343
[illegible]	.8[illegible]	.0953	2.2036	2.1083

CONFIDENCE MULTIPLIERS FOR
K = 16.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .90

COB	Z	Z1	Z2	Z3
.6100	.7996	.1078	2.1934	2.[illegible]
.6200	.7829	.1193	2.1774	2.[illegible]
.6300	.7664	.1310	2.1647	2.[illegible]
.6400	.7500	.1423	2.1521	2.[illegible]
.6500	.7336	.1534	2.1395	1.9665
.6600	.7177	.1643	2.1273	1.9635
.6700	.7016	.1749	2.1160	1.9411
.6800	.6860	.1853	2.1044	1.9190
.6900	.6703	.1955	2.0923	1.8974
.7000	.6547	.2055	2.0817	1.8751
.7100	.6391	.2153	2.0706	1.8553
.7200	.6236	.2249	2.0597	1.8348
.7300	.6082	.2343	2.0489	1.8146
.7400	.5927	.2435	2.0383	1.7946
.7500	.5774	.2525	2.0279	1.7753
.7600	.5620	.2614	2.0176	1.7562
.7700	.5465	.2701	2.0074	1.7374
.7800	.5311	.2786	1.9974	1.7188
.7900	.5156	.2870	1.9875	1.7006
.8000	.5000	.2952	1.9778	1.6826
.8100	.4843	.3032	1.9681	1.6649
.8200	.4685	.3111	1.9586	1.6475
.8300	.4526	.3189	1.9492	1.6303
.8400	.4364	.3265	1.9399	1.6134
.8500	.4201	.3340	1.9307	1.5967
.8600	.4035	.3414	1.9216	1.5803
.8700	.3866	.3486	1.9126	1.5640
.8800	.3693	.3557	1.9037	1.5480
.8900	.3516	.3627	1.8949	1.5322
.9000	.3333	.3695	1.8862	1.5167

CONFIDENCE MULTIPLIERS FOR
K = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COH	I	II	III	IV
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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CONFIDENCE MULTIPLIERS FOR
K = 10.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COH	I	II	III	IV
.3100	1.4919	[illegible]	[illegible]	[illegible]
.3200	1.4577	[illegible]	[illegible]	[illegible]
.3300	1.4249	[illegible]	[illegible]	[illegible]
.3400	1.3933	-.3050	[illegible]	[illegible]
.3500	1.3626	[illegible]	[illegible]	[illegible]
.3600	1.3333	[illegible]	2.5405	2.7961
.3700	1.3049	[illegible]	2.5166	2.7464
.3800	1.2773	-.2098	2.4936	[illegible]
.3900	1.2506	-.1882	2.4713	2.6595
.4000	1.2247	-.1674	2.4498	2.6172
.4100	1.1996	-.1473	2.4290	2.5763
.4200	1.1751	-.1279	2.4089	2.5367
.4300	1.1513	-.1091	2.3894	2.4985
.4400	1.1282	-.0909	2.3705	2.4614
.4500	1.1055	-.0732	2.3521	2.4254
.4600	1.0835	-.0562	2.3343	2.3905
.4700	1.0619	-.0396	2.3170	2.3566
.4800	1.0408	-.0235	2.3002	2.3237
.4900	1.0202	-.0079	2.2836	2.2917
.5000	1.0000	.0073	2.2675	2.2605
.5100	.9802	.0220	2.2517	2.2300
.5200	.9606	.0364	2.2370	2.2006
.5300	.9417	.0504	2.2222	2.1716
.5400	.9230	.0640	2.2078	[illegible]
.5500	.9045	.0773	2.1936	2.1164
.5600	.8864	.0902	2.1798	2.0896
.5700	.8686	.1026	2.1663	2.0635
.5800	.8510	.1151	2.1531	2.0379
.5900	.8336	.1271	2.1401	[illegible]
.6000	.8165	.1389	2.1274	[illegible]

CONFIDENCE MULTIBLIRAS FOR
R = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COR	P	T1	T2	[illegible]
.6100	.7946	.1963	[illegible]	1.4547
.6200	.7839	.1615	[illegible]	1.4415
.6300	.7664	[illegible]	[illegible]	[illegible]
.6400	.7500	.1832	2.0791	[illegible]
.6500	.7336	.1937	2.0676	[illegible]
.6600	.7177	.2039	[illegible]	[illegible]
.6700	.7018	.2139	2.0452	1.8312
.6800	.6860	.2236	2.0342	1.8195
.6900	.6703	.2334	2.0235	1.7981
.7000	.6547	.2424	2.0129	1.7781
.7100	.6391	.2520	2.0025	1.7504
.7200	.6236	.2611	1.9922	1.7312
.7300	.6082	.2700	1.9822	1.7122
.7400	.5927	.2787	1.9722	1.6935
.7500	.5774	.2872	1.9624	1.6752
.7600	.5620	.2956	1.9528	1.6572
.7700	.5465	.3038	1.9432	1.6394
.7800	.5311	.3119	1.9338	1.6220
.7900	.5156	.3198	1.9246	1.6048
.8000	.5000	.3275	1.9154	1.5879
.8100	.4843	.3352	1.9064	1.5712
.8200	.4686	.3427	1.8975	1.5548
.8300	.4528	.3500	1.8888	1.5386
.8400	.4364	.3572	1.8799	1.5227
.8500	.4201	.3643	1.8713	1.5069
.8600	.4035	.3713	1.8628	1.4914
.8700	.3866	.3782	1.8543	1.4762
.8800	.3693	.3849	1.8460	1.4611
.8900	.3516	.3916	1.8377	1.4462
.9000	.3333	.3980	1.8295	1.4315

CONFIDENCE MULTIBLIRAS FOR
R = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COR	P	T1	T2	[illegible]
.0100	9.9499	[illegible]	[illegible]	[illegible]
.0200	7.0000	[illegible]	[illegible]	[illegible]
.0300	[illegible]	[illegible]	[illegible]	[illegible]
.0400	4.8990	[illegible]	[illegible]	[illegible]
.0500	4.3589	[illegible]	[illegible]	[illegible]
.0600	3.9591	[illegible]	[illegible]	[illegible]
.0700	3.6456	-1.[illegible]	[illegible]	[illegible]
.0800	3.3919	-1.6795	[illegible]	[illegible]
.0900	3.1746	-1.[illegible]	[illegible]	[illegible]
.1000	2.9833	-1.[illegible]	[illegible]	[illegible]
.1100	2.8449	-1.[illegible]	[illegible]	[illegible]
.1200	2.7080	-1.1415	[illegible]	[illegible]
.1300	2.5869	-1.[illegible]	[illegible]	[illegible]
.1400	2.4765	-.9630	3.1677	[illegible]
.1500	2.3805	-.8870	3.1114	[illegible]
.1600	2.2915	-.8171	3.0421	[illegible]
.1700	2.2096	-.7532	2.9765	[illegible]
.1800	2.1344	-.6974	2.9207	[illegible]
.1900	2.0647	-.6442	2.8670	[illegible]
.2000	2.0000	-.5948	2.8173	[illegible]
.2100	1.9396	-.5490	2.7710	[illegible]
.2200	1.8829	-.5061	2.7275	[illegible]
.2300	1.8297	-.4660	2.6873	[illegible]
.2400	1.7795	-.4284	2.6492	[illegible]
.2500	1.7321	-.3939	2.6133	[illegible]
.2600	1.6871	-.3594	2.5794	[illegible]
.2700	1.6443	-.3277	2.5472	[illegible]
.2800	1.6036	-.2976	2.5167	[illegible]
.2900	1.5647	-.2691	2.4877	[illegible]
.3000	1.5275	-.2419	2.4601	[illegible]

[illegible] MULTIPLIERS FOR
[illegible] DEGREES OF FREEDOM
[illegible] COEFFICIENT = [illegible]

[illegible]	[illegible]	T1	T2	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	-.0441	2.2579	[illegible]
[illegible]	[illegible]	-.0261	2.2394	[illegible]
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[illegible]	1.1513	.0241	2.1874	[illegible]
[illegible]	1.1282	.0399	2.1711	2.1517
[illegible]	1.1055	.0551	2.1551	[illegible]
[illegible]	1.0835	.0699	2.1400	2.0701
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[illegible]	1.0408	.0981	2.1106	[illegible]
[illegible]	1.0202	.1116	2.0965	1.9949
[illegible]	1.0000	.1247	2.0827	1.9761
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[illegible]	.9417	.1619	2.0436	1.8817
[illegible]	[illegible]	.1737	2.0311	[illegible]
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[illegible]	.8864	.1965	2.0071	1.8108
[illegible]	.8686	.2075	1.9955	1.7882
[illegible]	.8510	.2179	1.9841	1.7660
[illegible]	[illegible]	[illegible]	1.9730	1.7447
[illegible]	[illegible]	[illegible]	1.9621	1.7236

CONFIDENCE MULTIPLIERS FOR
N = 24.[illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COR	L	T1	T2	[illegible]
.6100	.7996	[illegible]	[illegible]	[illegible]
.6200	[illegible]	[illegible]	[illegible]	[illegible]
.6300	.7684	.2676	[illegible]	[illegible]
.6400	.7500	[illegible]	[illegible]	[illegible]
.6500	.7331	[illegible]	[illegible]	[illegible]
.6600	.7177	.2948	[illegible]	[illegible]
.6700	.7011	[illegible]	[illegible]	[illegible]
.6800	.6860	[illegible]	[illegible]	[illegible]
.6900	.6703	[illegible]	[illegible]	[illegible]
.7000	.6547	[illegible]	[illegible]	[illegible]
.7100	.6391	[illegible]	[illegible]	[illegible]
.7200	.6236	[illegible]	[illegible]	[illegible]
.7300	[illegible]	[illegible]	[illegible]	[illegible]
.7400	.5927	.3596	[illegible]	[illegible]
.7500	[illegible]	.3673	[illegible]	[illegible]
.7600	.5620	.3746	[illegible]	[illegible]
.7700	.5465	.3817	1.8041	[illegible]
.7800	.5311	[illegible]	[illegible]	[illegible]
.7900	.5156	[illegible]	1.7881	[illegible]
.8000	.5000	[illegible]	[illegible]	[illegible]
.8100	[illegible]	[illegible]	1.7725	[illegible]
.8200	.4685	[illegible]	[illegible]	[illegible]
.8300	.4526	[illegible]	[illegible]	[illegible]
.8400	.4364	[illegible]	[illegible]	[illegible]
.8500	.4201	[illegible]	[illegible]	[illegible]
.8600	.4035	.4408	[illegible]	[illegible]
.8700	.3866	.4468	1.7280	[illegible]
.8800	.3693	.4527	1.7208	[illegible]
.8900	.3516	.4586	1.7138	[illegible]
.9000	.3333	.4643	1.7066	[illegible]

[illegible]

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[illegible]	[illegible]	-1.[illegible]	[illegible]	[illegible]
.1100	[illegible]	-.[illegible]	[illegible]	3.[illegible]
.1200	[illegible]	-.8151	[illegible]	3.[illegible]
.1300	[illegible]	-.[illegible]	[illegible]	3.[illegible]
.1400	2.4785	-.6603	[illegible]	3.[illegible]
.1500	2.3805	-.5967	2.7564	3.[illegible]
.1600	2.2913	-.5369	2.6484	3.[illegible]
.1700	2.2096	-.4563	2.5454	3.[illegible]
.1800	2.1344	-.4179	[illegible]	[illegible]
.1900	2.0647	-.3933	[illegible]	[illegible]
.2000	2.0000	-.3420	[illegible]	[illegible]
.2100	1.9396	-.3135	2.4716	2.7[illegible]
.2200	1.8829	-.2777	2.4354	2.7131
.2300	1.8297	-.2441	2.4015	2.[illegible]
.2400	1.7795	-.2126	[illegible]	2.[illegible]
.2500	1.7321	-.1829	2.3336	2.[illegible]
.2600	1.6871	-.1548	2.3113	2.4[illegible]
.2700	1.6443	-.1263	2.2844	2.4127
.2800	1.6036	-.1031	2.2569	2.[illegible]
.2900	1.5647	-.0792	2.2346	2.[illegible]
.3000	1.5275	-.0564	2.2115	2.[illegible]

[illegible]

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.3900	1.[illegible]	.1[illegible]	[illegible]	1.[illegible]
.4000	1.[illegible]	.1[illegible]	[illegible]	1.[illegible]
.4100	1.1996	.1[illegible]	[illegible]	1.[illegible]
.4200	1.1751	.1[illegible]	1.[illegible]	1.[illegible]
.4300	1.1513	.1[illegible]	1.[illegible]	1.[illegible]
.4400	1.1282	.17[illegible]	1.[illegible]	1.[illegible]
.4500	1.1055	.1[illegible]	1.[illegible]	1.[illegible]
.4600	1.0835	.2048	1.9441	1.[illegible]
.4700	1.0619	.2166	1.9[illegible]	1.7[illegible]
.4800	1.0408	.22[illegible]	1.9[illegible]	1.[illegible]
.4900	1.0202	.2396	1.9[illegible]	1.[illegible]
.5000	1.0000	.25[illegible]	1.[illegible]	1.[illegible]
.5100	.9802	.2615	1.[illegible]	1.[illegible]
.5200	.9608	.2719	1.[illegible]	1.[illegible]
.5300	.9417	.2[illegible]	1.[illegible]	1.[illegible]
.5400	.9230	.29[illegible]	1.[illegible]	1.[illegible]
.5500	.9045	.30[illegible]	1.[illegible]	1.[illegible]
.5600	.8864	.3110	1.[illegible]	1.[illegible]
.5700	.8686	.3201	1.[illegible]	1.[illegible]
.5800	.8510	.3291	1.[illegible]	1.[illegible]
.5900	.8336	.3378	1.[illegible]	1.[illegible]
.6000	.8165	.3464	1.[illegible]	1.[illegible]

[illegible]
[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	.3866	[illegible]	[illegible]	1.0764
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.1000	[illegible]	[illegible]	[illegible]	[illegible]
.1100	2.8445	[illegible]	[illegible]	[illegible]
[illegible]	2.7060	-.4732	[illegible]	[illegible]
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.1400	2.4785	[illegible]	[illegible]	[illegible]
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.1700	2.2096	-.2120	2.3166	[illegible]
.1800	2.1344	-.1730	2.2775	[illegible]
.1900	2.0647	-.1370	2.2414	[illegible]
.2000	2.0000	-.1037	2.2079	[illegible]
.2100	1.9396	-.0727	2.1767	[illegible]
.2200	1.8829	-.0438	2.1474	[illegible]
.2300	1.8297	-.0167	2.1200	[illegible]
.2400	1.7795	.0088	2.0944	[illegible]
.2500	1.7321	.0327	2.0704	[illegible]
.2600	1.6871	.0553	2.0475	[illegible]
.2700	1.6443	.0766	2.0259	[illegible]
.2800	1.6036	.0971	2.0057	[illegible]
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[illegible]

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[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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CONFIDENCE MULTIPLIERS FOR
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 .1500   2.3805   -1.8551   4.4114   6.2664
 .1600   2.2913   -1.7462   4.3019   6.0441
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0800	5.3912	-2.8900	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0900	3.1798	-2.6413	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.1000	3.0000	-2.4304	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.1100	2.8445	-2.2485	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.1200	2.7080	-2.0895	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.1300	2.5869	-1.9469	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9302	.1400	2.4785	-1.8207	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9415	.1500	2.3805	-1.7093	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9711	.1600	2.2913	-1.6065	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9806	.1700	2.2096	-1.5136	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9806	.1800	2.1344	-1.4265	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9406	.1900	2.0647	-1.3470	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9305	.2000	2.0000	-1.2723	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9211	.2100	1.9396	-1.2048	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.9116	.2200	1.8829	-1.1406	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2300	1.8297	-1.0809	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2400	1.7795	-1.0246	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.8637	.2500	1.7321	-.9716	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.8747	.2600	1.6871	-.9215	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.8616	.2700	1.6443	-.8741	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2800	1.6036	-.8291	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.8453	.2900	1.5647	-.7864	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.8397	.3000	1.5275	-.7457	[illegible]	[illegible]

CONFIDENCE MULTIPLIERS FOR
N = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

[illegible]	[illegible]	C1	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.1000	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.1400	[illegible]	[illegible]	[illegible]	[illegible]
.1500	[illegible]	[illegible]	[illegible]	[illegible]
.1600	[illegible]	[illegible]	[illegible]	[illegible]
.1700	[illegible]	[illegible]	[illegible]	[illegible]
.1800	[illegible]	[illegible]	[illegible]	[illegible]
.1900	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	2.1017
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.2500	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	1.8944
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

CONFIDENCE MULTIPLIERS FOR
N = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

[illegible]	[illegible]	C1	[illegible]	[illegible]
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[illegible]	1.4577	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	1.2773	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.2801	[illegible]	[illegible]
.4000	[illegible]	[illegible]	[illegible]	[illegible]
.4100	[illegible]	[illegible]	[illegible]	[illegible]
.4200	1.1751	[illegible]	[illegible]	[illegible]
.4300	1.1513	.3261	[illegible]	[illegible]
.4400	1.1282	.3367	[illegible]	[illegible]
.4500	1.1055	[illegible]	[illegible]	[illegible]
.4600	1.0835	[illegible]	[illegible]	[illegible]
.4700	1.0619	[illegible]	[illegible]	[illegible]
.4800	1.0408	[illegible]	[illegible]	[illegible]
.4900	1.0202	.3849	[illegible]	[illegible]
.5000	1.0000	.3937	[illegible]	[illegible]
.5100	.9802	.4023	[illegible]	[illegible]
.5200	.9608	.4107	[illegible]	[illegible]
.5300	.9417	.4189	[illegible]	[illegible]
.5400	.9230	.4268	[illegible]	[illegible]
.5500	.9045	.4345	[illegible]	[illegible]
.5600	.8864	.4421	[illegible]	[illegible]
.5700	.8686	.4495	[illegible]	[illegible]
.5800	.8510	.4567	[illegible]	[illegible]
.5900	.8336	.4637	[illegible]	[illegible]
.6000	.8165	.4706	[illegible]	[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

CONFIDENCE MULTIPLIERS FOR
[illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .950

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.6071	[illegible]	[illegible]
[illegible]	[illegible]	.6116	1.4440	[illegible]
[illegible]	.6547	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.6236	.6247	1.4301	.8054
[illegible]	[illegible]	[illegible]	[illegible]	.7957
[illegible]	.5927	.6331	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.6411	[illegible]	.7714
[illegible]	[illegible]	.6451	[illegible]	.7633
[illegible]	[illegible]	.6489	[illegible]	.7552
[illegible]	[illegible]	.6527	1.4001	.7473
[illegible]	.5000	[illegible]	[illegible]	.7395
[illegible]	[illegible]	[illegible]	1.3920	.7319
[illegible]	[illegible]	[illegible]	1.3881	.7243
[illegible]	[illegible]	.6673	[illegible]	.7166
[illegible]	.4364	.6709	[illegible]	.7095
.6500	.4201	.674	1.3766	.7022
.6600	[illegible]	.6777	[illegible]	.6951
.6700	.3866	.6811	1.3691	.6880
.6800	[illegible]	.6844	1.3654	.6811
.6900	[illegible]	.6877	[illegible]	.6742
.7000	[illegible]	.6909	[illegible]	.6674

CONFIDENCE MULTIPLIERS FOR
K = 172.0 DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = .950

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.0400	4.8990	-.4646	2.4971	[illegible]
.0500	4.3589	-.3050	2.3376	[illegible]
.0600	3.9581	-.1666	2.2194	[illegible]
.0700	3.6450	-.0946	2.1272	2.2216
.0800	3.3912	-.0202	2.0527	[illegible]
.0900	3.1798	.0416	1.9908	1.9492
.1000	3.0000	.0940	1.9383	1.8449
.1100	2.8445	.1393	[illegible]	1.7537
.1200	2.7080	.1788	1.8534	1.6746
.1300	2.5869	.2138	1.8184	1.6046
.1400	2.4785	.2450	1.7871	1.5420
.1500	2.3805	.2732	1.7589	1.4857
.1600	2.2913	.2987	1.7333	1.4346
.1700	2.2096	.3220	1.7099	1.3879
.1800	2.1344	.3434	1.6885	1.3451
.1900	2.0647	.3631	1.6687	1.3056
.2000	2.0000	.3814	1.6503	1.2690
.2100	1.9396	.3984	1.6333	1.2349
.2200	1.8829	.4142	1.6173	1.2031
.2300	1.8297	.4291	1.6024	1.1733
.2400	1.7795	.4431	1.5884	1.1453
.2500	1.7321	.4562	1.5752	1.1190
.2600	1.6871	.4686	1.5627	1.0941
.2700	1.6443	.4804	1.5509	1.0705
.2800	1.6036	.4915	1.5396	1.0481
.2900	1.5647	.5021	1.5290	1.0269
.3000	1.5275	.5122	1.5188	1.0066

COEFFICIENT MULTIPLIERS FOR
K = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	SAM	T	T1	T2	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3300	1.4377	.3361	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3400	1.4244	.3507	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3500	1.3935	.3617	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3600	1.3626	.3744	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3800	1.3333	.3864	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3900	1.3049	.3981	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3900	1.2773	.4084	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3900	1.2500	.4183	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4000	1.2247	.4259	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4100	1.1996	.4351	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4200	1.1751	.4440	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4300	1.1513	.4527	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4400	1.1282	.4610	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4500	1.1055	.4692	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4600	1.0835	.4773	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4700	1.0619	.4847	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4800	1.0408	.4921	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4900	1.0202	.4993	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5000	1.0000	.5063	[illegible]	[illegible]
[illegible]	1.3396	[illegible]	[illegible]	[illegible]	.5100	.9802	.5131	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5200	.9608	.5197	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5300	.9417	.5262	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5400	.9230	.5325	[illegible]	[illegible]
[illegible]	1.7321	[illegible]	[illegible]	[illegible]	.5500	.9045	.5387	[illegible]	[illegible]
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[illegible]	1.6449	[illegible]	1.7982	[illegible]	.5700	.8686	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5800	.8510	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5900	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.6000	[illegible]	[illegible]	[illegible]	[illegible]

[illegible] MULTIPLIERS FOR
[illegible] DEGREES OF FREEDOM
[illegible] COEFFICIENT = [illegible]

[illegible]	D	T1	T2	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.3850	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	1.1966	.2824	[illegible]	[illegible]
[illegible]	1.1751	.3037	1.7951	1.4414
[illegible]	1.1513	.3147	1.7839	1.4[illegible]
[illegible]	1.1282	.3253	1.7730	1.4376
[illegible]	1.1055	.3357	1.7624	1.4267
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[illegible]	1.0405	.3648	1.7324	1.3976
[illegible]	1.0202	.3739	1.7229	1.3890
[illegible]	[illegible]	.3828	1.7137	1.3[illegible]
.8101	.9802	.3915	1.7047	1.3143
[illegible]	.9606	.3999	1.6960	1.2961
[illegible]	.9417	.4081	1.6875	1.2793
.5472	.9230	.4161	1.6791	1.2630
.5[illegible]	.9045	.4239	1.6710	1.2471
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CONFIDENCE MULTIPLIERS FOR
[illegible] = [illegible] DEGREES OF FREEDOM
CONFIDENCE COEFFICIENT = [illegible]

COEF	D	T1	T2	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
.6400	.7000	.4604	[illegible]	[illegible]
.6500	.7330	[illegible]	[illegible]	[illegible]
.6600	.7177	[illegible]	[illegible]	[illegible]
.6700	.7018	[illegible]	[illegible]	[illegible]
.6800	.6860	.5106	[illegible]	[illegible]
.6900	.6703	.5166	[illegible]	[illegible]
.7000	.6547	.5219	[illegible]	[illegible]
.7100	.6391	[illegible]	[illegible]	[illegible]
.7200	.6236	[illegible]	[illegible]	[illegible]
.7300	.6082	[illegible]	[illegible]	[illegible]
.7400	.5927	.5484	.5441	[illegible]
.7500	.5774	.5484	[illegible]	[illegible]
.7600	.5620	[illegible]	[illegible]	[illegible]
.7700	.5465	[illegible]	[illegible]	[illegible]
.7800	.5311	[illegible]	[illegible]	[illegible]
.7900	.5156	[illegible]	[illegible]	[illegible]
.8000	.5000	.5729	[illegible]	[illegible]
.8100	.4845	.5775	[illegible]	[illegible]
.8200	.4685	[illegible]	[illegible]	[illegible]
.8300	.4526	[illegible]	[illegible]	[illegible]
.8400	.4364	[illegible]	[illegible]	[illegible]
.8500	.4201	[illegible]	[illegible]	[illegible]
[illegible]	.4035	[illegible]	[illegible]	[illegible]
.8700	.3866	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

APPENDIX D

TABLE GENERATING PROGRAM

Description of Program CTAB2

Program CTAB2 calls subroutines SEARCH, BISECT, FSTAR, and PGISS to compute the constants t and t . CTAB2 is simply a driver routine with which to set up values to call SEARCH and then write results in organized tabular form. The four subroutines are provided with self-explanatory documentation. The one modification is that CTAB2 must multiply CH and CH from search by the parameter D from Equation (4.3) and divide by k/2 the number of spectral lines in the averaging. This is true because the subroutine FSTAR computes the values for a random variable with characteristic function

$$\phi(t) = (1 - it/k + t^2)^{-k/2}$$

The modification gives a variable with characteristic function

$$\phi(t) = \left(1 - \frac{it}{k/2} + \frac{t^2 D}{k^2/4}\right)^{-k/2}$$

which is that of equation (3.2) with = k/2 the desired form.

```
      PROGRAM CTAB2(INPUT,OUTPUT,TAPE2)
      DIMENSION TAB(90,5),RN(9)
      DATA (RN(I),I=1,9) /48.,10.,8.,9.,12.,17.,26.,42.,56./
      A=.05
      CC=1.-A
      NT=90
      RNT=NT
      DT=.9/RNT
      DO 20 K=1,9
      PRINT*,RN(K)
      RK=2*RN(K)
      DO 10 I=1,NT
      COH=(I)*DT
      DEL=SQRT(COH/(1.-COH))
      D=1./DEL
      CALL SEARCH(DEL,RN(K),A,CL,CU)
      TAB(I,1)=COH
      TAB(I,2)=D
      TAB(I,3)=CL*D/RN(K)
      TAB(I,4)=CU*D/RN(K)
      TAB(I,5)=TAB(I,4)-TAB(I,3)
   10 CONTINUE
      WRITE(2,91)
      WRITE(2,92)RK
      WRITE(2,95)CC
      WRITE(2,93)
      DO 30 I=1,30
   30 WRITE(2,94)(TAB(I,J),J=1,5)
      WRITE(2,91)
      WRITE(2,92)RK
      WRITE(2,95)CC
      WRITE(2,93)
      DO 40 I=31,60
   40 WRITE(2,94)(TAB(I,J),J=1,5)
      WRITE(2,91)
      WRITE(2,92)RK
      WRITE(2,95) CC
      WRITE(2,93)
      DO 50 I=61,90
   50 WRITE(2,94)(TAB(I,J),J=1,5)
      PRINT*,"RK=",RK
   20 CONTINUE
   91 FORMAT("")
   92 FORMAT(7X,"CONFIDENCE MULTIPLIERS FOR",/,7X,"K = ",
     &F5.1," DEGREES OF FREEDOM")
   93 FORMAT(7X,"COH",7X,"D",9X,"T1",8X,"T2",8X,"W")
   94 FORMAT(1X,5F10.4)
   95 FORMAT(7X,"CONFIDENCE COEFFICIENT = ",F5.3)
      STOP
```

```
      SUBROUTINE SEARCH(D,RN,A,CL,CU)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     THIS SUBROUTINE SEARCHES FOR STARTING POINTS FOR                C
C     THE SUBROUTINE BISECT.                                          C
C     VARIABLES:                                                      C
C                D=NONCENTRALITY PARAMETER                            C
C                NU=DEGREES FREEDOM                                   C
C                A=SIGNIFICANCE LEVEL                                 C
C                CL=LOWER CONFIDENCE LIMIT                            C
C                CU=UPPER CONFIDENCE LIMIT                            C
C                                                                     C
C     SEARCH COMPUTES A/2 AND 1-A/2 THEN FINDS POINTS                 C
C     THAT HAVE VALUES OF THE FUNCTION FSTAR THAT                     C
C     STRADDLE A/2 AND 1-A/2. BISECT IS THEN CALLED                   C
C     TWICE TO ZERO IN ON THE LOWER AND UPPER CONFIDENCE              C
C     LIMITS. BY N. ANDREW UNIV. OF KY.                               C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      ITC=0
      A2=A/2.0
      A3=1.-A2
      IF(RN.LE.0.) PRINT*,"ILLEGAL RN"
      IF(RN.LE.0.) RETURN
      T=D*RN
   10 F=FSTAR(T,D,RN)
      IF(F.LE.A2.OR.F.GE.A3)GO TO 50
      TU=T
      TL=T
      RINC=(.1)*SQRT((D**2+2.0)*RN)
   20 TLS=TL
      TL=TL-RINC
      FL=FSTAR(TL,D,RN)
      IF(FL.LT.A2)GO TO 30
      GO TO 20
   30 TUS=TU
      TU=TU+RINC
      FU=FSTAR(TU,D,RN)
      IF(FU.GT.A3)GO TO 40
      GO TO 30
   40 CALL BISECT(TL,TLS,CL,A2,D,RN)
      CALL BISECT(TUS,TU,CU,A3,D,RN)
      RETURN
   50 IF(F.LE.A2) T=T+RINC
      IF(F.GE.A3) T=T-RINC
      ITC=ITC+1
        IF(ITC.GT.1)PRINT*,"TROUBLE",ITC
      GO TO 10
      RETURN
      END
```

```
      SUBROUTINE BISECT(TL,TU,T,F0,D,RN)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     BISECT USES THE BISECTION METHOD TO ZERO IN ON              C
C     A VALUE T THAT CORRESPONDS WITH A VALUE OF THE              C
C     FUNCTION FSTAR(T)=F0.                                       C
C     VARIABLES:                                                  C
C                TL=LOWER GUESS AT T PASSED FROM                  C
C                   SUBROUTINE SEARCH                             C
C                TU=UPPER GUESS AT T PASSED FROM                  C
C                   SUBROUTINE SEARCH                             C
C                T=FINAL APPROXIMATION TO T                       C
C                F0=VALUE OF FUNCTION FSTAR THAT                  C
C                   FSTAR(T) MUST BE CLOSE TO                     C
C                D=NONCENTRALITY PARAMETER                        C
C                RN=DEGREES FREEDOM                               C
C     BY M. ANDREW UNIV. OF WY.                                   C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      TOL=.00001
      STD=SQRT((D**2+2.0)*RN)
   10 IF((ABS(TU-TL)/STD).LT.TOL) GO TO 20
      TLS=TL
      TL=(TL+TU)/2.0
      F=FSTAR(TL,D,RN)
      IF(F.LT.F0) GO TO 10
      TU=TL
      TL=TLS
      GO TO 10
   20 T=(TL+TU)/2.0
      RETURN
      END
```

```
      FUNCTION PCTAR(W,D,RNU)
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     PCTAR COMPUTES THE VALUE P(VAR.LT.W) WHERE VAR          C
C     REFERS TO ANY VARIABLE WITH CHARACTERISTIC              C
C     FUNCTION G(T)=1/(1-I*D*T-T**2)**RNU WHERE               C
C     I=SQRT(-1.0).                                           C
C     VARIABLES:                                              C
C               W=UPPER LIMIT OF INTEGRATION                  C
C               D=NONCENTRALITY PARAMETER                     C
C               RNU=DEGREES FREEDOM                           C
C     PCTAR USES THE FACT THAT INTEGRALS OF THE GAMMA         C
C     TYPE CAN BE WRITTEN AS POISSON SUMS (SEE CH.IV).        C
C     BY L. ANDREW UNIV. OF WY.                               C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      RN=1.0
      NUM=RNU
      R1=(D+SQRT(D**2+4.))/2.0
      R2=(D-SQRT(D**2+4.))/2.0
      R3=SQRT(D**2+4.)
      PCTAR=0.0
      IF(W.LT.0.0)GO TO 20
      DO 10 I=1,NUM
      J=I-1
      XJ=J
      IDIF=RNU+XJ
      IDIF=RNU-XJ
      Z=-R1*W
      EI=1.0/(R1**IDIF)+(1.0-POISS(Z,IDIF))/(R2**IDIF)
      PCTAR=PCTAR+RN*EI/(R3**ISUM)
      RN=RN*(RNU+XJ)/(XJ+1.)
   10 CONTINUE
      RETURN
      DO 20 I=1,NUM
      J=I-1
      XJ=J
      IDIF=RNU+XJ
      ISUM=RNU-XJ
      Z=-R1*W
      EI=1.0/(R1**IDIF)+(1.0-POISS(Z,IDIF))/(R2**IDIF)
      PCTAR=PCTAR+RN*EI/(R3**ISUM)
      RN=RN*(RNU+XJ)/(XJ+1.)
```

```
      FUNCTION POISS(U,M)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     POISS DIRECTLY COMPUTES THE POISSON SUM FROM ZERO              C
C     TO M OF THE POISSON PROBABILITY FUNCTION WITH                  C
C     PARAMETER U.                                                   C
C     BY D. ANDREW UNIV. OF WY.                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      POISS=0.0
      TERM=1.0
      DO 10 I=1,M
      RX=I
      POISS=POISS+TERM
      TERM=TERM*U/RX
   10 CONTINUE
      IF(U.LT.600.0)A=EXP(-U)
      IF(U.GE.600.0)A=0.
      POISS=POISS*A
      RETURN
      END
```

END

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